Stepping Stones or Traps?

The Dynamics of Informal Employment

Jose M. Quintero*

November 2, 2026

[Link to latest version]

Abstract

Does access to informal employment help workers entering the labor market reach formal employment? I show that informality provides a stepping stone while lengthening the route and lowering the probability of reaching formality. It serves as a stepping stone by providing income, human capital accumulation, and lower search costs than in unemployment. Yet its value weakens incentives to search intensively and incur formal-entry costs. To quantify this trade-off, I estimate a model of heterogeneous workers' search, sectoral choices, and human capital dynamics using long histories of formal and informal employment from Chile. Removing the informal sector raises new workers' probability of reaching formality by 14 percentage points and expands the tax base. However, the private loss from eliminating this employment option outweighs the fiscal gain, lowering welfare for workers entering the labor market.

*Email: jmquintero925@uchicago.edu. I am grateful to my advisors, Fernando Alvarez, Robert Shimer, Chang-Tai Hsieh, and Rodrigo Adão, for their consistent guidance. I also thank Dylan Balla-Elliott, Juanma Castro-Vincenzi, Craig A. Chikis, Santiago Franco, Mikhail Golosov, Greg Kaplan, Santiago Lacouture, Sreyas Mahadevan, Isaac Norwich, Ryungha Oh, Esteban Rossi-Hansberg, Marcos Sora, Harald Uhlig, and Jun Wong for insightful comments. This research uses data from the Encuesta de Protección Social. I thank the Subsecretaría de Previsión Social, which holds the survey's intellectual property rights, for permission to use the anonymized database. All results are my responsibility and do not represent the views of the Subsecretaría.

1 Introduction

Informal employment accounts for between 20 and 80 percent of the labor force in low- and middle-income countries (Bobba et al., 2025). Informality broadly encompasses employment relationships that do not comply with labor regulations. Formal employment offers the prospect of higher wages and faster wage growth for workers as well as a broader tax base for government. Yet informal employment provides income while avoiding some of the entry costs and recurring burdens of formality.[1] Workers sort across sectors by weighing the prospect of higher formal earnings against these costs. But the benefits of being informal today may come at the cost of never reaching formal employment. In this paper, I answer the question of whether access to informal employment helps workers entering the labor market reach formal employment.

I find that informal employment provides a stepping stone to formality while lengthening the route and lowering the probability of reaching it. It serves as a stepping stone by allowing workers to earn income, build human capital, and search for formal employment at a lower cost than in unemployment. Yet the value of being informal reduces the gain from formalizing, weakening incentives both to search intensively and to incur the costs of formal entry. Despite delaying formal entry, informal employment has social value. Hence, removing the informal option lowers average welfare for new workers.

To establish these results, I develop a model in which heterogeneous workers choose how intensively to search for formal and informal employment as their human capital evolves. The model maps workers' unobserved human capital and search decisions into predictions about transitions between labor-market states and the duration of spells, defined as uninterrupted periods in a given state. I estimate the model using long labor histories of both formal and informal workers collected by Chile's workers' survey, the Encuesta de Protección Social (EPS). Finally, I use the estimates to compare new workers' probability of reaching formality during their life cycle across economies with and without an informal sector and quantify the social value of the informal option.

In Section 2, I show how the gains from formal employment can change while a worker remains informal. Workers differ in human capital, which determines their earnings in both sectors. I build on the work of Lucas & Prescott (1974), allowing both unemployed and employed workers to search across formal and informal labor markets, represented as islands. Formal and informal employment offer different returns to human capital, while formal employment carries a recurring cost independent of human capital. Human capital evolves differently across the three labor-market states, changing workers' earnings and their incentives to move between them.

The model explains the duration of informal spells through three mechanisms. First, higher search costs reduce search effort and can prolong informal spells. I allow these costs to differ between informal and unemployed workers. Second, entry costs reduce search effort and can deter search altogether when they exceed the gain from formalizing. Finally, human capital dynamics

[1] Meghir et al. (2015) and Franco & Quintero (2026) document differences in wages and wage growth between formal and informal employment. Studies of the costs of formality include de Mel et al. (2013) on registration costs, Zárate (2026) on commuting frictions, and Olken et al. (2024) and Bernal et al. (2025) on fiscal costs.

can change that gain as a spell unfolds, strengthening or weakening incentives to search and move. Together, these mechanisms shape both the time workers spend in informality and their prospects of ever reaching formal employment.

To learn the strength of these mechanisms, I turn to the labor histories documented in Section 3. Most panels that include informal workers are short, making it difficult to study how informality shapes workers' careers.[2] The EPS reconstructs an average of 23 years of employment and unemployment for both formal and informal workers. I use these histories to estimate the formal-entry hazard, a worker's transition probability into formal employment conditional on elapsed informal spell duration. I find that this probability declines with time spent in informality, from 7.8 percent at the start of a spell to about 2 percent after five years. The model shows that this decline can reflect changing incentives within a spell as well as selection among workers who remain informal. I estimate the model to distinguish these explanations and assess how informal employment affects workers' formal-entry prospects.

In Section 4, I estimate the model by jointly matching transition patterns, wages, and employment shares. Hazard moments constrain how human capital dynamics and selection combine to change mobility during a spell. Wages provide complementary information about workers' human capital, with the model accounting for who sorts into each sector and how long they remain there. Transition destinations and employment shares further constrain the frictions and payoffs governing search and sectoral choices. I use this joint evidence to quantify the mechanisms that support formal entry and those that sustain informal employment.

The estimated parameters in Section 4 explain how informality provides a stepping stone to formality while lengthening the path to it. The recurring cost of formal employment makes it unattractive to workers with low human capital, who sort into informality. Once informal, workers face lower costs of searching for formality than unemployed workers. Raising these costs to their unemployment counterpart lowers new workers' average probability of reaching formality by 3.91 percentage points.[3] Yet high entry costs, combined with the smaller gain from leaving informality, reduce search effort and delay formal entry. Removing the entry cost raises this probability by 14.56 percentage points. By contrast, human capital accumulation helps workers progress toward formality. Its rate nearly matches that in formal employment, although informal workers are exposed to idiosyncratic shocks to human capital. Setting the informal accumulation rate equal to its unemployment counterpart lowers formal access by 4.20 percentage points. Overall, conditional on reaching formality, workers in the economy with informality take longer to enter but spend less time unemployed on average than those in the economy without it. The informal option has private value because it allows workers to search for formality at lower cost, accumulate human capital, and earn income along the way.

Finally, in Section 5, I show that removing the informal sector raises new workers' probability of reaching formality by 14.08 percentage points, but the average welfare effect is -17.02 percent.

[2]Studies using short panels include Bosch & Maloney (2010) and Cano-Urbina (2015).

[3]These exercises isolate one mechanism at a time while holding workers' other choices fixed. Removing the informal sector instead allows workers to adjust all their choices.

This welfare loss arises because the private losses outweigh the fiscal gains from a broader tax base. I therefore evaluate how formalization policies can expand access while preserving the benefits of informal employment. I compare three different policies: greater oversight of informality, a lower formal tax wedge, and a subsidy to the recurring cost of formal employment. For each policy, I choose the level that delivers the same gain in formal access. Among these three policies, only the subsidy raises both output and average welfare for new workers. Easing these recurring burdens can thus expand formal access while retaining the informal option.

Literature Review. This paper contributes to three strands of literature. The first studies the determinants of informality, mostly through the decisions of firms. As Bobba et al. (2025) review, much of the quantitative work models firms that decide whether to register, whom to hire off the books, and how large to grow when formality is costly (La Porta & Shleifer, 2014; Ulyssea, 2018). Unlike these papers, I study informality from the side of workers, as a state that they enter and leave over their careers.

Second, I take the tools that the macro-labor literature uses to study labor-market transitions (see Castro et al., 2024, for a review) and apply them to a setting in which workers move in and out of informality. In developing economies, high labor-market flows reflect a slippery job ladder, as Donovan et al. (2023) document.[4] Alvarez et al. (2024) separate structural duration dependence from dynamic selection in a stopping-time model. I build on their work with a model of informality in which human capital moves continuously and workers choose where to search. The model distinguishes changes in workers' incentives within a spell from changes in the composition of workers who remain informal.

Finally, I contribute to the literature on workers' selection into informal jobs and the lasting effects of informal employment. Magnac (1991) contrasts a segmented market, in which barriers restrict access to formal jobs, with a competitive one, in which workers sort by comparative advantage. Meghir et al. (2015) explain sorting through search frictions, arguing that frequent moves between sectors are difficult to reconcile with permanent comparative advantage. López García (2015), Finamor (2025), and Bobba et al. (2022) explain sorting through differences in comparative advantage, wealth, or schooling. In my model, the least skilled sort into informality by comparative advantage, while entry costs hold back workers with a comparative advantage in formal employment. Sorting is dynamic: workers' comparative advantage and search incentives evolve within employment spells.

Informal employment can also have lasting effects through job ladders (Seminario-Amez, 2021) or differences in human capital growth across sectors (Franco & Quintero, 2026). Closest to this paper, Bobba et al. (2021) combine search frictions with human capital that grows faster on formal jobs. In their setting, however, workers' incentives to search and move do not change as a spell unfolds. In my model, human capital dynamics change workers' comparative advantage during a spell. Their interaction with search and entry costs allows me to assess whether informality serves

[4]Their paper leaves open how selection interacts with labor-market frictions, which they point to as an avenue for future theories.

as a stepping stone to formal employment or remains a persistent employment state.

2 The Model

In this section, I develop a model of how workers move across unemployment, informal employment, and formal employment. Human capital determines each worker's comparative advantage across sectors, while differences in search costs shape how intensively workers search given the gain from switching. That gain, however, depends on the worker's current state: if informal employment is sufficiently valuable relative to unemployment, the gain from formalization may not cover the entry cost. Human-capital dynamics can then widen or narrow the gain from formalization as a spell unfolds, strengthening or weakening workers' incentives to formalize. I first set up the environment and the equilibrium, and then study how these three mechanisms shape where workers search and how their exit rates evolve over a spell.

2.1 Production and Wages

Time is continuous, and production takes place on islands that differ only in their status, formal or informal. For each status, there is a unit mass of islands indexed by $j \in [0, 1]$. All islands share the same productivity and produce a homogeneous final good, which trades freely across islands and serves as the numeraire.[5]

Production uses labor supplied by workers who differ only in their human capital. A worker with human capital x supplies x efficiency units of labor. A worker is formally employed on a formal island, informally employed on an informal island, and unemployed when not on an island. An island j of status $\ell \in \{i, f\}$ produces

$$y_{\ell j}(t) = A \int_0^\infty x \, n_{\ell j}(x, t) \, \mathrm{d}x, \tag{1}$$

where $n_{\ell j}(x, t)$ is the employment of workers with human capital x on the island and $A > 0$ is a common productivity. The technology in equation (1) exhibits constant returns to scale in efficiency units of labor, and hence the marginal product for every worker depends only on their own human capital, not on who else works there.

Formal and informal islands differ only in the wedge between a worker's marginal product and their wage. On formal islands, wages carry a proportional tax wedge τ. On informal islands, employment avoids the tax, but a fraction $\alpha_0 \in [0, 1)$ of output is lost to concealment and other costs of evading enforcement. Unlike the tax, this loss is not a transfer to the government but a deadweight cost for the economy. Since labor markets within islands are competitive, each worker

[5] Competitive islands with frictional mobility across them follow Lucas & Prescott (1974) and Alvarez & Shimer (2011). In those papers, islands differ in product demand or in productivity, whereas here they differ only in status. Although a continuum is not technically necessary for the baseline model, it preserves the interpretation of search as finding employment across many local labor markets.

earns their marginal product net of these wedges:

$$w_f(x) = \frac{A}{1+\tau}\, x, \qquad w_i(x) = A(1-\alpha_0)\, x. \tag{2}$$

Both wages are proportional to human capital, $w_\ell(x) = a_\ell x$, where a_ℓ is the return to human capital on islands of status ℓ. Two properties of these wages shape workers' choices: First, formal employment pays more per unit of human capital when the enforcement loss exceeds the tax wedge, $\alpha_0 > \tau/(1+\tau)$. Second, neither wage depends on how workers are allocated across islands, since the marginal product of each worker does not depend on who else works there.[6] Workers therefore take wages as given when they decide where to search. All islands of the same status pay the same wages, so a worker cares only about the status of their island. I focus on symmetric allocations within each status and refer to these islands collectively as sector ℓ. With a unit mass of islands per status, employment density on each island equals the sector employment density. I therefore drop the island index and write this density as $n_\ell(x,t)$.

2.2 Workers

The economy contains a unit mass of workers, who search across these islands for employment at the wages each island pays, discount the future at rate $\rho > 0$, and leave the labor force at Poisson rate $\delta > 0$. Each departing worker is immediately replaced by a new unemployed worker, whose human capital is drawn from a distribution $G(x)$ with continuous density $g(x)$ on $\mathbb{R}_+$. At each point in time, a worker's labor market state, $\ell \in \{u, i, f\}$, records whether they are unemployed or employed on an informal or a formal island.

Workers are risk neutral, cannot save, and supply labor inelastically, so their only choices are where to search and when to quit. Each worker also receives a common lump-sum transfer $\mathcal{T}(t)$, which, given linear utility, leaves these choices unchanged.[7] An unemployed worker receives government benefits $b \geq 0$, while a worker with human capital x employed in sector $\ell \in \{i, f\}$ earns the wage $w_\ell(x)$. Formal workers also bear a recurring nonpecuniary cost $F > 0$, which summarizes compliance requirements and other non-wage burdens of formality.

Transition Frictions. Workers cannot move at will to the island they prefer, because three frictions slow their movement across labor market states. First, moving into employment requires search. A worker in state $\ell \in \{u, i, f\}$ chooses how hard to search for islands of each status $\ell' \in \{i, f\} \setminus \{\ell\}$, and their search effort delivers opportunities on those islands at rate $\eta_\ell^{\ell'}(x) \geq 0$.[8] Searching harder, however, is increasingly costly, and the cost depends on where the worker searches

[6] A natural extension lets enforcement losses grow with the size of an informal island, since larger informal islands are easier for the government to detect. Informal production then exhibits decreasing returns to scale, and, as in Lucas & Prescott (1974), wages depend not only on workers' human capital but also on the size of their island, so they must be solved jointly with the distribution of workers.

[7] Because the transfer shifts all state values equally, I suppress it until the equilibrium definition.

[8] The feasible search links are $u \to i$, $u \to f$, $i \to f$, and $f \to i$.

from:

$$\psi_\ell^{\ell'}(\eta_\ell^{\ell'}) = \frac{\kappa_\ell^{\ell'}}{2}\left(\eta_\ell^{\ell'}\right)^2, \qquad \kappa_\ell^{\ell'} > 0.$$

The parameter $\kappa_\ell^{\ell'}$ captures the search cost that a worker in ℓ faces when looking for islands of status ℓ'. Because this cost varies by origin, the same type of island can be easier to reach from one state than from another.

Second, joining a formal island requires a one-time entry cost $\Psi_f \geq 0$, which captures paperwork, legal requirements, and other costs of formalization, while informal islands carry no such cost, $\Psi_i = 0$. A worker therefore moves into formality only when the gain covers the entry cost, but moves into informality whenever the gain is positive.

Third, employment ends exogenously. Informal and formal employment spells end at rates δ_i and δ_f, sending workers into unemployment. When these rates differ, the two sectors offer different stability, which workers weigh alongside wages. Not every exit from employment is involuntary, however. Workers may also quit once unemployment is at least as valuable as their current employment, which can happen when their human capital is low and their wage is close to the unemployment benefit.

Human Capital. At a given level of human capital, these payoffs and frictions fix each worker's incentives across states. Within a spell, fixed human capital would leave the worker's search incentives unchanged. Human capital instead evolves according to a process that depends on the worker's current state:

$$\mathrm{d}x_t = \mu_\ell \,\mathrm{d}t + \sigma_\ell \,\mathrm{d}W_t, \qquad \ell \in \{u, i, f\},$$

where W_t is a standard Brownian motion, and human capital cannot fall below zero.[9]

In each state, human capital grows or depreciates at rate μ_ℓ. Unemployed workers lose human capital, $\mu_u < 0$, while formal workers build it, $\mu_f > 0$. For informal employment, I impose only that it is no worse than unemployment and no better than formal employment for accumulation, $\mu_u \leq \mu_i \leq \mu_f$.[10] This restriction accommodates three possibilities: informal workers may lose human capital no faster than unemployed workers, preserve it, or build it no faster than formal workers. Workers also experience idiosyncratic shocks to their human capital, whose volatility $\sigma_\ell > 0$ may differ across states.

Value Functions and HJB Equations. Where to search and when to quit depend on how much each state is worth to a worker. Let $U(x)$, $V_i(x)$, and $V_f(x)$ denote the values of unemployment, informal employment, and formal employment for a worker with human capital x. Since every worker enters the labor force unemployed, I start with the value of unemployment, which

[9] The full process is $\mathrm{d}x_t = \mu_\ell \,\mathrm{d}t + \sigma_\ell \,\mathrm{d}W_t + \mathrm{d}L_t$, where L_t is the minimal nondecreasing process that keeps $x_t \geq 0$ and increases only when $x_t = 0$.

[10] The upper bound is in line with the faster wage growth of formal workers documented by Franco & Quintero (2026).

satisfies the Hamilton–Jacobi–Bellman (HJB) equation

$$\begin{aligned}\rho U(x) = b + \mu_u \partial_x U(x) + \tfrac{\sigma_u^2}{2}\partial_{xx} U(x) - \delta U(x) \\ + \max_{\eta_u^i, \eta_u^f \geq 0} \left\{ \sum_{\ell \in \{i,f\}} \left[\eta_u^\ell(x) \left[V_\ell(x) - U(x) - \Psi_\ell \right] - \psi_u^\ell\left(\eta_u^\ell(x)\right) \right] \right\}.\end{aligned} \tag{3}$$

Equation (3) values unemployment as the benefit a worker collects while waiting plus the option of finding employment. While waiting, the worker collects the benefit b, but their human capital fluctuates with shocks and drifts down, which lowers the value of waiting. Waiting also ends at rate δ, when the worker leaves the labor force. Meanwhile, the worker generates opportunities in each sector and moves there when one arrives. They choose each search intensity by weighing the gain from the destination, net of its entry cost, against the cost of generating opportunities.

Once employed, a worker faces two margins that unemployment lacks, exogenous separations and the option to quit back into unemployment. The value of employment therefore satisfies a variational inequality,

$$\begin{aligned}\rho V_\ell(x) \geq w_\ell(x) - \mathbb{1}_f F + \mu_\ell \partial_x V_\ell(x) + \tfrac{\sigma_\ell^2}{2}\partial_{xx} V_\ell(x) - \delta V_\ell(x) \\ + \delta_\ell \left[U(x) - V_\ell(x) \right] + \max_{\eta_\ell^{-\ell} \geq 0} \left\{ \eta_\ell^{-\ell} \left[V_{-\ell}(x) - V_\ell(x) - \Psi_{-\ell} \right] - \psi_\ell^{-\ell}\left(\eta_\ell^{-\ell}\right) \right\},\end{aligned} \tag{4}$$

subject to $V_\ell(x) \geq U(x)$, where $-\ell$ denotes the other sector.

The employment equation follows the same logic as the unemployment HJB. An informal worker earns $w_i(x)$, and a formal worker earns $w_f(x)$ net of the recurring cost F. While employed, the worker's human capital keeps drifting and fluctuating, and they can search for employment in the other sector. Employment also ends at rate δ_ℓ, when a separation sends them back to unemployment.

Because an employed worker can always quit, the value of employment cannot fall below the value of unemployment, $V_\ell(x) \geq U(x)$. When $V_\ell(x) > U(x)$, the worker stays employed, and the employment HJB in equation (4) holds as an equality. When $V_\ell(x) = U(x)$, the worker is indifferent between employment and unemployment and quits. The indifference point $\bar{x}_u^\ell$ separates workers who quit from those who stay.

The same equations determine how hard workers search. For $\ell \in \{i, f\}$, optimal search effort is

$$\eta_u^\ell(x) = \frac{[V_\ell(x) - U(x) - \Psi_\ell]^+}{\kappa_u^\ell}, \qquad \eta_\ell^{-\ell}(x) = \frac{[V_{-\ell}(x) - V_\ell(x) - \Psi_{-\ell}]^+}{\kappa_\ell^{-\ell}}, \tag{5}$$

Search responds on two margins. On the extensive margin, a worker searches toward a destination only when the gain from reaching it exceeds its entry cost. On the intensive margin, they search harder when this net gain is larger and when their search cost is lower. With $\Psi_f > 0$, the

extensive margin binds for some workers who would rather be formal, since their gain from formal employment is positive but does not cover the entry cost.

These search rules shape a central choice for an unemployed worker who would rather be formal: whether to accept informal employment in the meantime. Informality provides a wage and preserves the option to search for formality, but changes search incentives in three ways. First, the search cost becomes κ_i^f rather than κ_u^f, so the same gain can induce more or less intensive search. Second, becoming formal now means giving up informal employment rather than unemployment. The gain therefore falls from $V_f - U$ to $V_f - V_i$ and may no longer cover the entry cost Ψ_f. Third, human capital follows the informal rather than the unemployment process, changing how the gain from formalization evolves during the spell. Informality can therefore strengthen formal search, weaken it, or eliminate the incentive to search.

2.3 Distributional Dynamics

Individual choices add up to a distribution of workers across islands and unemployment, which, unlike values and policies, can change over time. Let $m_\ell(x,t)$ denote the density of workers with human capital x in state $\ell \in \{u, i, f\}$ at time t. Four forces move workers across this distribution: changes in human capital, search, separations and quits, and labor-force turnover.

First, accumulation, depreciation, and shocks move workers up and down the human-capital distribution while they remain in the same labor market state. For a density m of workers over human capital, the net number of workers in state ℓ who rise past human-capital level x per unit of time is

$$\mathcal{J}_\ell[m](x,t) = \mu_\ell \, m(x,t) - \frac{\sigma_\ell^2}{2} \partial_x m(x,t). \tag{6}$$

The first term captures accumulation when $\mu_\ell > 0$ and depreciation when $\mu_\ell < 0$. The number of workers crossing x depends on their rate of human-capital accumulation or depreciation, μ_ℓ, and on the density at x. The second term captures shocks and depends on $\partial_x m_\ell(x,t)$, the change in density with human capital. Shocks are symmetric for individuals, but more workers cross from the more populated side of x, generating a net flow toward less populated levels.

These flows change the density through their difference across nearby human-capital levels. A narrow band gains workers when the net upward flow at its lower end exceeds that at its upper end, and loses workers in the opposite case. Human-capital movements therefore contribute $-\partial_x \mathcal{J}_\ell[m_\ell](x,t)$ to the change in density at x.

Second, search moves workers across labor market states. A worker in state ℓ with human capital x finds an opportunity on an island of status ℓ' at rate $\eta_\ell^{\ell'}(x)$, so the number of workers moving from ℓ to ℓ' at x per unit of time is $\eta_\ell^{\ell'}(x) m_\ell(x,t)$. Since workers carry their human capital with them, the same term is an outflow from ℓ and an inflow into ℓ' at the same x. This holds whether the worker moves from unemployment onto an island or between islands.

Third, employment ends through separations and quits. Workers in sector $\ell \in \{i, f\}$ lose their

employment at rate δ_ℓ, which generates an outflow $\delta_\ell m_\ell(x,t)$ from that sector and an equal inflow into unemployment. Workers also quit, leaving employment once their human capital falls to the indifference point $\bar{x}_u^\ell$. Let $q_\ell(x,t)$ denote this quitting flow. It occurs only at the indifference point, where it equals the net number of workers who fall past that point, $-\mathcal{J}_\ell[m_\ell](\bar{x}_u^\ell, t)$, and it is zero when quitting is absent. Like separations, quits move workers into unemployment at their current human capital.

Finally, labor-force exit and replacement change the composition of the population. Workers exit the labor force at rate δ from every state, which removes $\delta m_\ell(x,t)$ from each density. Exiting workers are replaced one for one by entrants, so the labor force has the perpetual-youth structure of overlapping cohorts with a constant size. Entrants start unemployed with human capital drawn from $g(x)$, and since the population has unit mass, entry adds $\delta g(x)$ to unemployment.

Adding up the four forces gives the Kolmogorov forward equations, which track how the density of workers in each state changes over time:

$$\partial_t m_\ell(x,t) = \begin{cases} -\partial_x \mathcal{J}_u[m_u](x,t) - \big[\eta_u^i(x) + \eta_u^f(x) + \delta\big] m_u(x,t) + \delta g(x) \\ \qquad\qquad + \delta_i m_i(x,t) + \delta_f m_f(x,t) + q_i(x,t) + q_f(x,t), & \ell = u, \\ -\partial_x \mathcal{J}_i[m_i](x,t) - \big[\eta_i^f(x) + \delta_i + \delta\big] m_i(x,t) \\ \qquad\qquad + \eta_u^i(x) m_u(x,t) + \eta_f^i(x) m_f(x,t) - q_i(x,t), & \ell = i, \\ -\partial_x \mathcal{J}_f[m_f](x,t) - \big[\eta_f^i(x) + \delta_f + \delta\big] m_f(x,t) \\ \qquad\qquad + \eta_u^f(x) m_u(x,t) + \eta_i^f(x) m_i(x,t) - q_f(x,t), & \ell = f. \end{cases} \tag{7}$$

Because search, separations, and quits only move workers across states, and entry exactly offsets exit, the population remains constant at all times.[11] The densities also deliver the aggregates that determine the equilibrium. In particular, integrating each density over human capital gives the population share in that state, $\mathbf{M}_\ell(t)$.

2.4 Equilibrium

Government Budget. The government sets the tax wedge τ, the enforcement rate α_0, and unemployment benefits b, and each instrument acts on its budget through workers' choices. A higher tax wedge raises revenue per formal worker but lowers the return to formal employment, which can shrink the tax base. More generous benefits raise spending directly and make unemployment more attractive. Stricter enforcement raises no revenue of its own, but by lowering the return to informality it can push workers into the tax base. The lump-sum transfer $\mathcal{T}(t)$ returns whatever revenue is left after benefits, so the government's flow budget constraint is

$$\tau \int_0^\infty w_f(x) m_f(x,t)\,\mathrm{d}x = b\,\mathbf{M}_u(t) + \mathcal{T}(t), \tag{8}$$

[11] Appendix A.2 shows that the process preserves mass.

Since the population is normalized to one, the transfer each worker receives equals the aggregate transfer, and it becomes a lump-sum tax when benefits exceed revenue. The budget thus pins down the transfer at every instant, given the policy instruments and the distribution of workers.

In an equilibrium, every island's labor market clears at the competitive wages in equation (2), workers search and quit optimally at these wages, the distribution of workers evolves as their choices dictate, and the government balances its budget. The following definition states this formally.

Definition 1 (Equilibrium)**.** *An equilibrium consists of wages* $\{w_i(x), w_f(x)\}_{x\geq 0}$*, island employment* $\{n_i(x,t), n_f(x,t)\}_{x\geq 0}$*, worker distributions* $\{m_\ell(x,t)\}_{x\geq 0,\,\ell\in\{u,i,f\}}$*, lump-sum transfers* $\{\mathcal{T}(t)\}_{t\geq 0}$*, value functions* $(U(x), V_i(x), V_f(x))$*, and policy functions* $\left(\eta_u^i(x), \eta_u^f(x), \eta_i^f(x), \eta_f^i(x)\right)$*, such that:*

(i) Each island's labor market is competitive, so wages satisfy equation (2)*.*

(ii) Workers optimize: value functions and search policies satisfy equations (3)*–*(5)*.*

(iii) Worker distributions satisfy equation (7)*, starting from an initial distribution* $m_\ell(x,0)$ *and with the corresponding boundary conditions.*

(iv) Each island's labor market clears:

$$n_{\ell j}(x,t) = n_\ell(x,t) = m_\ell(x,t), \qquad \ell \in \{i,f\}, \tag{9}$$

and the population constraint

$$1 - \mathbf{M}_u(t) = \mathbf{M}_i(t) + \mathbf{M}_f(t)$$

holds.

(v) The government balances its budget according to equation (8)*.*

Stationary Equilibrium. I focus on a stationary equilibrium, in which individual workers keep moving across states and their human capital keeps changing, but the flows into and out of each level of human capital in each state balance. The distribution of workers, and with it the transfer, is then constant over time, and I drop the time argument from here on.

Two features of the model make the stationary equilibrium simple to characterize. First, wages do not depend on how workers are allocated, as Section 2.1 showed, so workers solve their problem at known wages and their choices never feed back into wages. Second, labor-force turnover pins down the distribution that these choices generate. Every worker is eventually replaced by an entrant, so the stationary distribution is an average over entrants' histories and does not depend on where the economy starts. The additional condition is that entrants have finite human capital on average. This ensures a finite stationary wage bill and lump-sum transfer. The following theorem summarizes these properties.

Theorem 1. *Suppose entrants' human capital has finite mean,* $\int_0^\infty x \, \mathrm{d}G(x) < \infty$. *Then a stationary equilibrium exists and is unique.*

Proof. See Appendix A.3.

2.5 Dynamic Sorting and Duration Dependence

In this section, I connect workers' human-capital dynamics and search decisions to observable wages and employment transitions. This mapping provides a basis for interpreting how informal employment shapes access to formality. I first examine how wages, recurring costs, and human-capital dynamics shape the value of each state. I then show how search and entry costs determine workers' responses to the gains from leaving their current state. Finally, I follow cohorts through their spells to explain how changes in individual search intensity and in the composition of survivors jointly shape observable destination-specific hazards.

Wages and Comparative Advantage. At a given human capital, a worker's gain from changing states depends on both the destination's value and the value of their current state. I call the employment sector with the higher lifetime value the worker's absolute advantage. Comparative advantage measures the gross value gain relative to their current state. Formal employment therefore has the absolute advantage when $V_f(x) > V_i(x)$, while its comparative advantage is $V_f(x) - U(x)$ from unemployment and $V_f(x) - V_i(x)$ from informal employment. These lifetime values depend in part on what workers receive and the recurring costs they bear in each state. By Equation (2), wages rise with human capital while the unemployment benefit b stays fixed. A worker who enters employment at low human capital may therefore give up benefits that exceed their wage. Between the two employment sectors, the formal wage premium, $a_f/a_i - 1$, measures the proportional difference in take-home wages at the same human capital. When this premium is positive, the wage gap $(a_f - a_i)x$ grows with human capital, while F remains fixed. The wage gap therefore fails to compensate for F at low human capital but does so at sufficiently high human capital.

Two dynamic forces build on these payoffs. First, employment stability raises the value of income earned while employed. A sector with a lower separation rate δ_ℓ allows the worker to collect their income for longer. Second, human-capital accumulation raises the value of employment, $V_\ell(x)$, because it raises the worker's wage while the spell lasts. To see how wages change during a spell, consider a worker who enters sector ℓ with human capital $x(0)$. At duration d, their wage has changed by

$$w_\ell(x(d)) - w_\ell(x(0)) = a_\ell \mu_\ell d + a_\ell \sigma_\ell W_d, \quad W_d \sim \mathcal{N}(0, d).$$

Here, W_d is the Brownian increment measured from spell entry. The first term captures wage growth or decline from accumulation or depreciation, while the second captures realized human-capital shocks.[12]

[12] This expression omits reflection at zero. When reflection occurs, add $a_\ell(L_d - L_0)$. Appendix D.1 discusses the

Human capital also affects the value of unemployment through future employment opportunities. A worker who never searches receives b until leaving the labor force. The expected net present value of these benefits is $b/(\rho + \delta)$ and does not depend on human capital. The option of finding employment can increase $U(x)$ above this benchmark, and its value depends on the wages the worker could earn. Employed workers, in turn, can always quit into unemployment, so employment is worth at least $U(x)$. At the indifference points $\bar{x}_u^\ell$, employment in sector $\ell \in \{i, f\}$ and unemployment have equal value. These options form part of the state values that determine workers' gains from changing states.

Optimal Search and Sorting. For workers who actively search for formal employment, a lower search cost can offset a smaller comparative advantage. Consider, for example, a worker with human capital $x > \bar{x}_u^i$. For this worker, $V_f(x) - V_i(x) < V_f(x) - U(x)$, since $V_i(x) > U(x)$. Their comparative advantage in formality is therefore smaller from informality than from unemployment. By Equation (5), they can nevertheless search more intensively from informality if κ_i^f is sufficiently low relative to κ_u^f.

A lower search cost cannot induce formal search unless the worker's comparative advantage exceeds the entry cost Ψ_f. As in the preceding example, consider an unemployed worker for whom $0 < V_f(x) - V_i(x) < \Psi_f < V_f(x) - U(x)$. At this human capital, the worker searches for formal employment from unemployment but not from informality, even though formal employment retains the absolute advantage. The search cost κ_ℓ^f can therefore offset the smaller gain from formality only on the intensive margin, and Ψ_f governs the extensive margin.

Human-capital accumulation adds a dynamic component to workers' search decisions. Set aside shocks during informal employment and suppose that $V_f(x) - V_i(x)$ increases with human capital. Consider again the worker who stops searching upon entering the informal sector. As this worker builds human capital, $\mu_i > 0$, their gain from formalization increases. The worker resumes formal search if this growing gain exceeds Ψ_f. By contrast, if human capital remains unchanged or depreciates during the spell, the gain stays fixed or falls, so formal search remains inactive.

Hazards and Duration Dependence. I now turn from workers' search decisions to destination-specific hazards, which connect unobserved search intensities to observable transition rates over a spell. As a spell unfolds, these hazards reflect both changes in individual search intensity and changes in the composition of the workers who remain.

Search frictions make sorting gradual, and because opportunities arrive at random, workers face competing risks. To see this, consider an unemployed worker who is searching for both formal and informal employment. Even if formal employment has the absolute advantage, they might first find an informal island and move to it. The worker's destination therefore depends on both their search decisions and which opportunity arrives first. Across workers, these exits can change the composition of those who remain unemployed.

corresponding boundary correction.

More explicitly, consider a cohort of workers who start a spell in state ℓ at the same time. The cohort follows the human-capital dynamics and exit rates in equation (7) but receives no inflows. A worker leaves the cohort upon moving to another state or exiting the labor force. For a destination ℓ' reached through search, the hazard at duration d is the average search intensity among workers still in the spell,

$$\lambda_\ell^{\ell'}(d) = \mathbb{E}_d[\eta_\ell^{\ell'}(x)], \tag{10}$$

where $\mathbb{E}_d$ denotes the expectation over the human-capital distribution $\bar{m}_\ell(x, d)$ of the members still in ℓ at duration d. If search policies did not depend on human capital, the destination-specific hazard would not change with duration.

Because search intensity depends on human capital, the cohort hazard can change with duration for two reasons. First, human-capital dynamics can change the search intensities of the workers who remain in the spell. Second, exits can change the composition of those workers, a force I refer to as dynamic selection.

To isolate this second force, hold each worker's human capital fixed, so that their search intensities remain constant throughout the spell. Individual exit rates are therefore constant. When workers differ in total search intensity, however, those who search more are more likely to leave sooner, so average total search among survivors falls. This is search selection, which accounts for all dynamic selection when quitting is absent. For a hazard between the two employment sectors, it takes the form

$$\partial_d \lambda_\ell^{\ell'}(d) = -\operatorname{Var}_d\big[\eta_\ell^{\ell'}(x)\big] \leq 0,$$

so the hazard declines whenever members of the cohort differ in search intensity.

The fixed-human-capital case also shows how the destinations of exits change as the spell progresses. To see this, consider a cohort of formal workers, and define

$$\chi_f^i(d) = \frac{\lambda_f^i(d)}{\lambda_f^i(d) + \lambda_f^u(d)} \tag{11}$$

as the share of exits from the spell at duration d that end in informal employment. With human capital fixed, workers do not quit, so $\lambda_f^u(d) = \delta_f$ remains constant. Search selection, by contrast, lowers $\lambda_f^i(d)$ whenever workers differ in informal search intensity. The informal share of exits $\chi_f^i(d)$ is therefore nonincreasing with duration.

When human capital changes during a spell, however, it can shift workers' comparative advantage and therefore their search intensities. I refer to this force as structural duration dependence. At the cohort level, these movements are described by the flux $\mathcal{J}_\ell[\bar{m}_\ell](x, d)$, obtained by applying equation (6) to the survivors' human-capital distribution.

Human-capital dynamics can also change the composition of the cohort by inducing quits into unemployment. Consider a worker in sector ℓ who quits when negative idiosyncratic shocks lower their human capital to the indifference point $\bar{x}_u^\ell$. To quantify this source of selection, let $\bar{q}_\ell(d)$

denote the quit rate among workers still in the cohort at duration d.[13] The following theorem decomposes changes in the hazard into structural duration dependence, search selection, and quit selection.

Theorem 2. *Consider a cohort in a stationary equilibrium. At almost every duration $d \geq 0$ with positive survivor mass, the unemployment hazards satisfy*

$$\partial_d \lambda_u^{\ell'}(d) = \underbrace{\int_0^\infty \partial_x \eta_u^{\ell'}(x) \mathcal{J}_u[\bar{m}_u](x,d)\,\mathrm{d}x}_{\substack{\text{structural duration} \\ \text{dependence}}} \underbrace{- \mathrm{Cov}_d\Big(\eta_u^{\ell'}(x), \eta_u^i(x) + \eta_u^f(x)\Big)}_{\text{search selection}}, \qquad \ell' \in \{i, f\}. \tag{12}$$

For an employment origin $\ell \in \{i, f\}$ with an upper continuation region, the hazard toward the other employment state satisfies

$$\partial_d \lambda_\ell^{\ell'}(d) = \underbrace{\int_{\bar{x}_u^\ell}^\infty \partial_x \eta_\ell^{\ell'}(x) \mathcal{J}_\ell[\bar{m}_\ell](x,d)\,\mathrm{d}x}_{\substack{\text{structural duration} \\ \text{dependence}}} \underbrace{- \mathrm{Var}_d[\eta_\ell^{\ell'}(x)]}_{\text{search selection}} \underbrace{+ \bar{q}_\ell(d)[\lambda_\ell^{\ell'}(d) - \eta_\ell^{\ell'}(\bar{x}_u^\ell)]}_{\text{quit selection}}. \tag{13}$$

When quitting is absent, the integral is over $\mathbb{R}_+$ and the quit-selection term is zero. Human capital is then reflected at zero.

Proof. See Appendix A.4.

Theorem 2 shows how human-capital movements affect the hazard through workers' search decisions. The structural term weights these movements, $\mathcal{J}_\ell[\bar{m}_\ell](x,d)$, by the slope of the search policy, $\partial_x \eta_\ell^{\ell'}(x)$. To see how it operates, set aside shocks, $\sigma_\ell = 0$, so the flux becomes $\mu_\ell \bar{m}_\ell(x,d)$. When workers search harder at higher human capital, accumulation strengthens their search and pushes the hazard up through the structural term, while depreciation weakens search and pushes it down. Where search intensity is flat, this contribution is zero.

Search selection depends on the origin. Employed workers search only toward the other sector, so their selection term is the negative variance from the fixed-human-capital example. Unemployed workers, however, can search toward both sectors, and those who search more in total leave through search at higher rates. To see how these departures affect either hazard, compare their search toward that destination with the average among survivors. If the departing workers search more toward it, their departure lowers that hazard through selection.

For employed cohorts, quit selection can work against search selection. Workers who reach the indifference point leave for unemployment. If they search less toward the other sector than the average survivor, their departure raises average search among those who remain and therefore the hazard toward that sector. Exogenous separations and labor-force exits also shrink the cohort, but because their rates do not depend on human capital, they do not change its composition.

[13] In a stationary equilibrium, the rate at which members of the cohort quit is precisely the net flow of workers at the indifference point, and thus, $\bar{q}_\ell(d) = -\mathcal{J}_\ell[\bar{m}_\ell](\bar{x}_u^\ell, d)$. If unemployment is dominated along the entire support of the human capital space, i.e., there is no quitting, then the reflecting barrier at zero implies that $\bar{q}_\ell(d) = 0$.

For an unemployment cohort, depreciation and search selection can reinforce each other. Suppose there are no idiosyncratic shocks and search intensity toward each sector weakly increases with human capital. As human capital depreciates, $\mu_u < 0$, search toward each sector can weaken, lowering the corresponding unemployment hazard through structural duration dependence. At the same time, workers who search more in total leave at higher rates. Their departures can also lower average search toward each sector among survivors. Corollary 1 formalizes why neither unemployment hazard increases with duration.

Corollary 1. *Consider a stationary equilibrium with $\sigma_u = 0$ in which $\partial_x \eta_u^{\ell'}(x) \geq 0$ for each $\ell' \in \{i, f\}$. For an unemployment cohort with finite mean initial human capital, $\partial_d \lambda_u^{\ell'}(d) \leq 0$ for each $\ell' \in \{i, f\}$ at almost every duration.*

Proof. See Appendix A.4.

The corollary, however, abstracts from idiosyncratic shocks, which can change individual search during unemployment. Positive shocks can raise a worker's human capital and strengthen search toward either sector. Thus, individual search need not weaken monotonically, and the corollary no longer guarantees weakly declining cohort hazards.[14]

Informal-to-Formal Mobility. Unlike the unemployment case above, informal employment need not align the two forces. To see this, again set aside idiosyncratic shocks and suppose that informal workers' search intensity toward formality is nondecreasing in human capital. Informality slows the depreciation of human capital, but that alone need not improve workers' later access to formality. If informal workers still lose human capital during the spell and quitting is absent, the logic of Corollary 1 applies. Structural duration dependence and dynamic selection then compound each other, so the hazard cannot increase. By contrast, if informal workers accumulate human capital, the two forces oppose each other, and the hazard depends on which one dominates. Even then, the hazard can decrease, as Corollary 2 shows.

Corollary 2. *Consider a stationary equilibrium in which $\sigma_i = 0$ and suppose that the comparative advantage of formality over informality increases with human capital so that*

$$\partial_x \eta_i^f(x) \geq 0,$$

with the inequality strict wherever $\eta_i^f(x) > 0$. For a new cohort of informal workers with heterogeneous initial formal-search intensities, and any finite duration window $[0, D]$, there is an explicit bound $\bar{\mu}_i(D) > 0$ on informal human capital accumulation such that $0 \leq \mu_i < \bar{\mu}_i(D)$ implies

$$\partial_d \lambda_i^f(d) < 0 \quad \text{for almost every } d \in [0, D].$$

Proof. See Appendix A.4.[15]

[14] Equation (5) shows that nondecreasing employment gains $V_\ell - U$ are sufficient for increasing unemployment search. Appendix A.1 explains when this fails and why it is a reasonable assumption.

[15] The proof also requires regularity conditions on the cohort's initial distribution, such as a finite mean and a

Theorem 2 separates the two forces behind this result. When informal workers accumulate human capital, their comparative advantage in formality grows, strengthening formal search wherever it is active. This pushes the hazard upward through structural duration dependence. Search selection pushes the other way because workers who search more intensively leave at higher rates. Corollary 2 shows that sufficiently slow accumulation leaves these individual increases in search too small to offset selection over the chosen duration window.

Idiosyncratic shocks affect the informal-to-formal hazard through changes in search intensity and through quitting. First, shocks move human capital up or down and can thereby alter search for formality. The structural contribution of shocks depends on the cohort's human-capital distribution and can have either sign. Second, negative shocks can bring workers to the indifference point $\bar{x}_u^i$, where they quit into unemployment. If quitters search less for formality than the average survivor, quit selection raises the informal-to-formal hazard and works against search selection.

A decreasing informal-to-formal hazard does not by itself imply that informality harms workers' future prospects of formalization. Corollary 2 shows that informal workers can accumulate human capital even as their cohort's hazard of moving to formality declines over the chosen duration window.

Taking Stock. Three mechanisms shape how informal employment affects a worker's access to formality. First, for active searchers, a lower search cost in informality can offset the smaller formalization gain and raise search intensity above its level in unemployment at the same human capital, as Equation (5) shows. Second, when an informal worker's gain from formalization does not exceed the entry cost, they do not search for formality even if formal employment has the absolute advantage. Finally, human-capital dynamics change the gain from formalization during the spell. Search resumes if these changes lift the gain above the entry cost.

The model maps these mechanisms into two observable objects: wages and destination-specific hazards. For wages, it links the formal premium to proportional differences in take-home pay across sectors at a given human capital, and within-spell wage changes to individual human-capital paths. By contrast, it decomposes changes in cohort hazards into individual search responses and selection among survivors, as Theorem 2 shows. Corollaries 1 and 2 then illustrate how structural duration dependence and search selection reinforce or oppose one another. These mappings motivate using wages and employment transitions jointly to assess how informal employment shapes access to formality.

3 Empirical Analysis

In the model, spell durations and exit destinations depend on which workers an employment state attracts, how their prospects evolve during a spell, and which of them leave first. Measuring these patterns requires following workers through complete spells, often over many years, which

share of workers who actively search for formal islands. The explicit bound $\bar{\mu}_i(D)$ is presented in the proof.

short panels cannot do (Bosch & Maloney, 2010). The EPS instead reconstructs each respondent's labor-market history retrospectively, often spanning decades. These histories serve two purposes: they recover statistical objects that short panels cannot measure, and they provide evidence on the model's mechanisms. I first describe the EPS and how its records map into the model's states, then document durations, hazards, and mobility, and finally add wages and previous employment to interpret these patterns through the model.

3.1 The EPS

The EPS is a longitudinal survey of Chilean adults that combines repeated interviews with retrospective labor-market histories. At each interview, respondents report the timing and characteristics of their employment and non-employment periods. Those who miss an interview can report the intervening months when they return.[16] I construct labor histories from the 2002, 2004, 2006, 2009, and 2015 rounds, excluding the 2012 reports because of interview problems and relying on the 2015 retrospective module to cover the missing years. Appendix B.1 documents the construction of the histories and their coverage across rounds, and Appendix B.2 documents the survey weights.

The analysis sample contains 23,205 respondents with at least one formal or informal employment spell. Table 1 contrasts survey participation with labor-history coverage: the average worker appears in three processed rounds but reports a history spanning 23 years, with similar coverage for men and women. These histories are also nearly continuous, with no gap longer than one month between consecutive spells for 95 percent of workers.[17]

Table 1: EPS sample overview

	Men	Women	All
Workers	12,474	10,731	23,205
Mean age at first interview	38.5	36.9	37.7
Average survey follow-up (rounds)	2.9	3.0	3.0
Average labor-history coverage (years)	23.1	23.3	23.2
Average net attrition per wave (%)	-8.4	-10.9	-9.5

Notes: The table describes respondents with at least one formal or informal employment spell. Worker counts are unweighted. Net attrition uses raw cross-sectional expansion weights, and negative values indicate growth in weighted respondent mass. Source: EPS 2002–2015 rounds; author's calculations.

Measuring Informality. Informality has no single empirical definition. Productive definitions classify jobs by features of the firm, whereas legal definitions classify them by the protection the worker receives, and the two need not agree (Perry et al., 2007). I adopt the legal view: an employee's job is formal only if the worker has a signed employment contract and contributes to

[16] The EPS follows respondents across rounds while periodically adding refreshment samples, so the interview panel is unbalanced and the time between a respondent's interviews varies.

[17] Weighted statistics use the respondent's survey expansion factor, normalized within the respondent's entry cohort and adjusted for differential response across rounds.

the pension or social-security system through that job.[18] This legal view includes arrangements that break the law, such as salaried work without mandated contributions, and arrangements that do not, such as self-employment outside the contributory system (Levy, 2008).

These requirements identify the jobs most likely to be subject to the model's tax wedge τ. Workers who contribute to the pension or social-security system are likely to appear in employer–employee administrative records through which payroll taxes are collected. A signed employment contract is legally required and provides similar evidence that the employment relationship is formally documented. I therefore require both indicators to classify an employee's job as formal, treating partial compliance as informality.[19]

The EPS records non-employment spells without distinguishing workers who look for a job from those who do not. I map these spells to the model's unemployment state, which allows workers to choose zero search effort. This mapping is more plausible for workers with strong labor-force attachment. Table 2 shows that women spend 46.5 percent of their labor histories out of work, more than twice the share for men (19.4 percent). Completed non-employment spells average 44.2 months for women, compared with 26.6 for men. I therefore restrict the duration and mobility analysis to men ages 15–64, among whom informal jobs account for 25.9 percent of employment.

Table 2: Labor-market attachment by sex

	Men	Women	All
Panel A. Time over the labor history (% of months)			
Formal employment	59.8	39.1	50.4
Informal employment	20.8	14.4	17.8
Non-employment	19.4	46.5	31.9
Panel B. Spells			
Employment spells per worker	5.0	4.1	4.6
Non-employment spells per worker	1.9	2.4	2.1
Mean completed non-employment spell (months)	26.6	44.2	35.4
Workers	12,447	10,713	23,160

Notes: Panel A reports the share of person-months spent in each state: weighted monthly shares, averaged across months from 1980 to 2016, using person-month stock weights. It covers 3,269,821 person-months of men and 2,882,520 of women. Panel B counts cleaned job-level spells that start at ages 15–64. Worker counts are unweighted. Ages assume a January birth month. Source: EPS 2002–2015 rounds; author's calculations.

[18] Article 9 of Chile's Labor Code requires a written employment contract signed by both parties but does not require notarization (Chile, 2003). Self-employed workers, including employers, have no employment contract, so I classify their jobs by contributions alone.

[19] Other studies weigh the two requirements differently: some use contributions alone, others a contract alone, and the narrowest definitions of informality require both to be absent (Perry et al., 2007; Bosch & Maloney, 2010). The gap between my definition and this narrower definition is the share of jobs that meet only one requirement.

3.2 Long-Run Dynamics of Informal Employment

I characterize long-run persistence and mobility using three objects: completed spell durations, destination-specific hazards, and spell-to-spell transition probabilities. To construct these objects, I organize labor histories into sector-spells that correspond to stays in the model's three states. Consecutive records belong to the same sector-spell when the state is unchanged and each begins within six months of the previous record's end. A change of state or a longer gap starts a new spell, while employer changes within a state do not. Each sector-spell enters the duration and mobility statistics once with a flow weight based on the worker's survey weights, so these statistics describe spell inflows rather than the stock of workers in each state. Appendix B.2 documents the construction, weights, and treatment of gaps and censoring.

Table 3: Sector-spell sample by employment state

	Formal	Informal	Unemployment
Number of sector-spells	24,115	13,633	23,506
Share of completed spells (%)	74.55	81.81	89.26
Mean completed duration (months)	61.92	42.16	26.64
Std. dev. of completed duration (months)	79.07	63.87	36.89
Range of completed durations (months)	1–437	1–432	1–356
Range of observed durations (months)	1–440	1–440	1–435
Workers	11,151	7,577	9,882

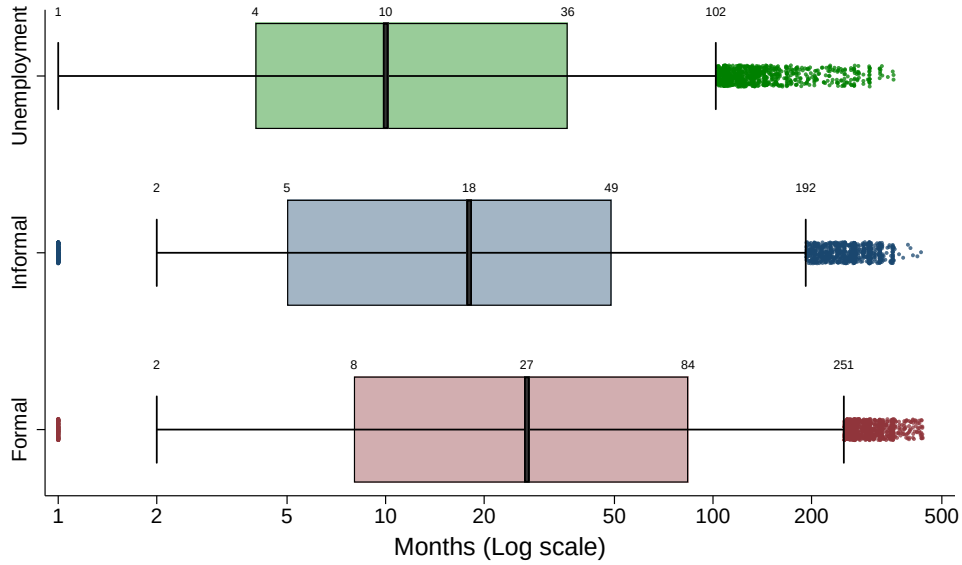
Notes: The table reports sector-spells of men ages 15–64 by origin state. A spell is completed when a later sector-spell is observed for the same worker. Spell counts and completion shares are unweighted, and duration moments use sector-spell flow weights. Source: EPS 2002–2015 rounds; author's calculations.

Spell Duration Distributions. The EPS histories record the end of most sector-spells in the sample. Table 3 describes 61,254 sector-spells from 12,537 men. A spell is completed when a later sector-spell is observed for the same worker, and its duration runs from its recorded start to its recorded end. Under this definition, the histories provide completed durations for 75 percent of formal spells and 89 percent of unemployment spells. These observations allow me to compare persistence across states over complete spells, including stays lasting many years.

Figure 1 shows that informal employment lies between formal employment and unemployment in persistence, with many short spells and a long upper tail. As Table 3 shows, completed informal spells last 42 months on average, compared with 62 for formal spells and 27 for unemployment spells. This ranking holds across the completed-duration distributions: formal durations first-order stochastically dominate informal durations, which in turn dominate unemployment durations. Half of completed informal spells end within 18 months, yet one in twenty lasts longer than 192 months. For some workers, informal employment therefore persists for many years.

Destination-Specific Hazards. Completed durations summarize how long workers remain in each state. Destination-specific hazards describe how their chances of moving to a particular

Figure 1: Completed spell duration distributions



Notes: The figure plots flow-weighted distributions of completed sector-spell durations for men ages 15–64. Boxes span the 25th to 75th percentiles, the thick line marks the median, whiskers span the 5th to 95th percentiles, and numbers above each box report these percentiles. Dots show individual spells beyond the whiskers, and Table 3 reports the full range. The sample contains 17,978,11,153, and 20,982 completed formal, informal, and unemployment spells. Source: EPS 2002–2015 rounds; author's calculations.

destination change as a spell unfolds. In the model, equation (10) defines this hazard as an instantaneous exit rate among workers still in the origin state. Because the EPS records spells in months, I estimate its discrete counterpart: the probability that a spell in progress at the start of a six-month interval ends with a move to that destination during the interval. Equation (C.1) relates these probabilities to the model's continuous-time hazards.

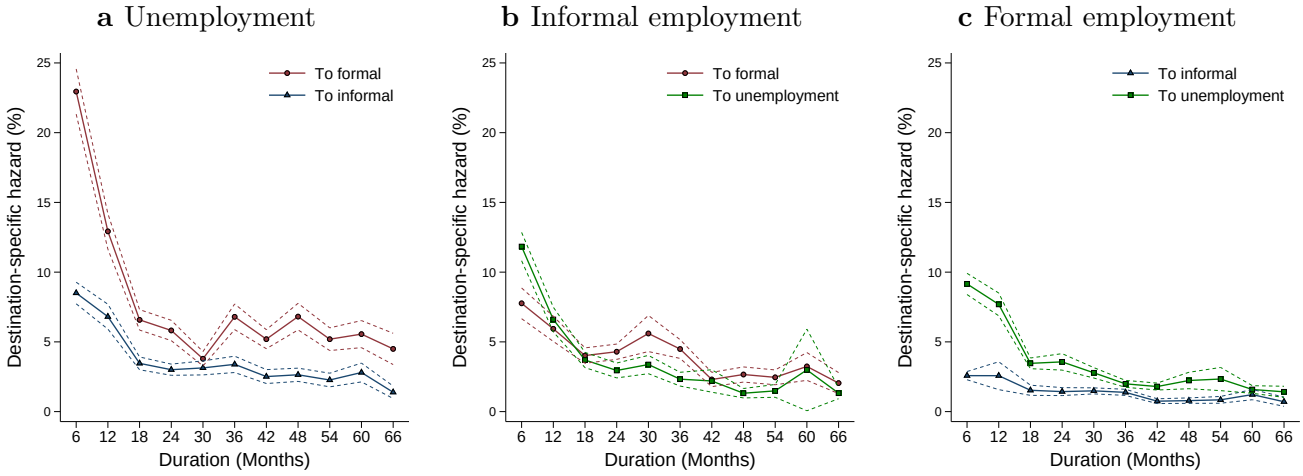
For each six-month interval, I estimate the hazard as the weighted number of exits to a particular destination divided by the weighted number of spells at risk at the interval's start.[20] For sector-spell n of worker m, let $\ell_{m,n} \in \{f, i, u\}$ denote its origin state, $T_{m,n}$ its observed duration in months, $\ell'_{m,n}$ its observed destination, and $\omega_{m,n}$ its flow weight. Over the interval $I_d = (d-6, d]$, the estimated hazard from ℓ to $\ell' \neq \ell$ is

$$\widehat{\lambda}_{\ell}^{\ell'}(d) = \frac{\sum_{m,n} \omega_{m,n} \mathbf{1}\{\ell_{m,n} = \ell,\ T_{m,n} \in I_d,\ \ell'_{m,n} = \ell'\}}{\sum_{m,n} \omega_{m,n} \mathbf{1}\{\ell_{m,n} = \ell,\ T_{m,n} > d-6\}}. \tag{14}$$

A spell still in progress when its history ends stays in the risk set through its last observed interval. When the next sector-spell begins more than six months later, or outside the three states (for example, in unpaid family work), I treat the destination as unknown. The spell leaves the risk set without counting toward either destination.

[20] The baseline intervals end at $d \in \{6, 12, \ldots, 66\}$ months. Appendix A.5 establishes worker-clustered inference.

Figure 2: Destination-specific hazards by origin state



Notes: Each panel plots the flow-weighted destination-specific hazard in Equation (14), in percent. The horizontal axis marks the end of each interval. The sample contains 23,506 unemployment,13,633 informal, and 24,115 formal sector-spells of men ages 15–64 from 12,537 workers. Dashed lines show pointwise 95% CIs based on worker-clustered standard errors. Source: EPS 2002–2015 rounds; author's calculations.

Figure 2 shows that exit probabilities in the first six months are highest from unemployment, intermediate from informal employment, and lowest from formal employment. First, unemployment spells end mostly in formal employment, with a hazard of 23.0 percent. Second, informal spells end more often in unemployment than in formal employment, with hazards of 11.8 and 7.8 percent, respectively. Finally, formal spells end mostly in unemployment, while direct moves into informality have a hazard of 2.6 percent. Thus, these initial exit probabilities are consistent with Figure 1's ordering of persistence, with informal employment between formal employment and unemployment.

These exit probabilities differ not only in their initial levels but also in how quickly they decline as spells lengthen, with most of the decline occurring in the first 18 months. From unemployment, the hazard into formal employment falls to an average of 5.5 percent between months 18 and 66. Among informal spells, the initially higher hazard into unemployment falls faster than the hazard into formal employment, leaving the two similar after the first year. Employment status alone therefore gives an incomplete account of mobility, which also varies sharply with elapsed spell duration.

Spell-to-Spell Mobility. To summarize where spells lead across durations, I estimate spell-to-spell transition probabilities. I calculate these probabilities directly as the weighted shares of linked exits from each origin that lead to each destination, using the full range of observed durations. Thus, they include exits after long spells that extend beyond the horizon shown in Figure 2.

Panel A of Table 4 shows that, when exits are pooled across durations, informal exits divide almost evenly between formal employment and unemployment, at 51 and 49 percent, respectively. By contrast, the other origins have a clear main destination, with 71 percent of formal exits leading to unemployment and 69 percent of unemployment exits leading to formal employment.

Table 4: Spell-to-spell mobility and duration

Panel A. Transition probability (%)				*Panel B. Median duration (months)*			
Origin	Formal	Informal	Unemployment	Origin	Formal	Informal	Unemployment
Formal	—	29.27	70.73	Formal	—	32	22
Informal	51.45	—	48.55	Informal	23	—	11
Unemployment	69.07	30.93	—	Unemployment	9	12	—

Notes: Rows are origin states and columns are destinations. Panel A reports the share of linked exits from each origin that lead to each destination, in percent. Panel B reports the weighted median duration of the origin spell, in months, for exits to that destination. The sample contains 47,396 linked exits of men ages 15–64 from 10,756 workers, and both panels use sector-spell flow weights. Source: EPS 2002–2015 rounds; author's calculations.

Thus, informal spells end in formal employment about as often as in unemployment.

Yet Panel B of Table 4 shows that these nearly equal shares of informal exits conceal a large difference in completed durations. Informal spells ending in formal employment last 23 months at the median, compared with 11 for those ending in unemployment. Similarly, formal spells ending in informality last 32 months at the median, compared with 22 for those ending in unemployment. By contrast, unemployment spells ending in formal employment are shorter than those ending in informality, with medians of 9 and 12 months, respectively. In both employment sectors, median completed durations are longer before job-to-job moves than before unemployment, providing evidence against a version of the model with no human capital dynamics.

In the absence of human capital dynamics, the model predicts the opposite duration ordering. Workers who search most intensively toward the other sector leave first, while the separation rate into unemployment remains constant. Consequently, the job-to-job share of exits in Equation (11) cannot increase with duration. This implies weakly shorter completed durations before job-to-job moves than before unemployment, including at the median. Yet Panel B shows longer spells before job-to-job moves in both sectors. These duration comparisons therefore contradict the model's fixed-human-capital benchmark.

Taken together, these moments provide complementary information about the model's mechanisms. Individually, however, they can be difficult to interpret because different combinations of mechanisms can generate similar patterns. An explanation of the data must therefore account jointly for spell durations, hazard profiles, and exit destinations. I next add wages and previous employment to this comparison, bringing further information about workers' differences and the incentives behind their transitions.

3.3 Wages and Duration Dependence

I now use wage evidence to help distinguish among explanations of the mobility patterns documented above. In the model, human capital determines both workers' earnings and their incentives to move between states. The same mechanisms must therefore explain both wages and mobility. However, the wage measure available from the EPS is the average over an employment spell. I

therefore compare these averages across completed spells of different lengths, without directly observing wage growth within each spell.

The relationship between average wages and completed duration confounds human capital dynamics with selection at spell entry. Longer spells allow more time for human capital to accumulate or depreciate, but workers may also enter these spells with different human capital. Human capital shocks can also change workers' search and quitting decisions, linking realized shocks to the duration of the spell. The covariance between average wages and completed duration therefore combines differences in initial human capital, accumulation or depreciation, and shock histories,

$$\operatorname{Cov}(T_\ell, \overline{w}_\ell) = a_\ell \Big[\operatorname{Cov}(T_\ell, x_0) + \frac{\mu_\ell}{2} \operatorname{Var}(T_\ell) + \sigma_\ell \operatorname{Cov}(T_\ell, \overline{W}_{T_\ell}) \Big]. \tag{15}$$

Here, T_ℓ is completed spell duration, x_0 is human capital at entry, and $\overline{W}_{T_\ell}$ is the average Brownian increment over the spell. Moreover, all covariances use the same completed-spell sample and flow weights as the wage-duration comparison, separately for each sector.[21]

Regressing average wages on completed spell duration shows that the formal–informal wage gap widens with duration. Figure 3a presents separate fits by sector for spells that begin out of unemployment and last at most 36 months: among the shortest of these spells, average wages are similar across sectors. However, as duration increases, the fitted formal wage profile rises more steeply than the informal profile.[22]

The steeper formal profile can reflect accumulation reinforced by selection at spell entry, as captured by Equation (15). Human capital accumulation raises workers' wages while they remain formally employed. If lower-wage formal entrants gain more from moving to informality, they search more intensively and tend to leave sooner. Workers with longer completed spells can then have higher human capital at entry. This selection would make the first covariance positive, steepening the formal profile alongside wage growth.

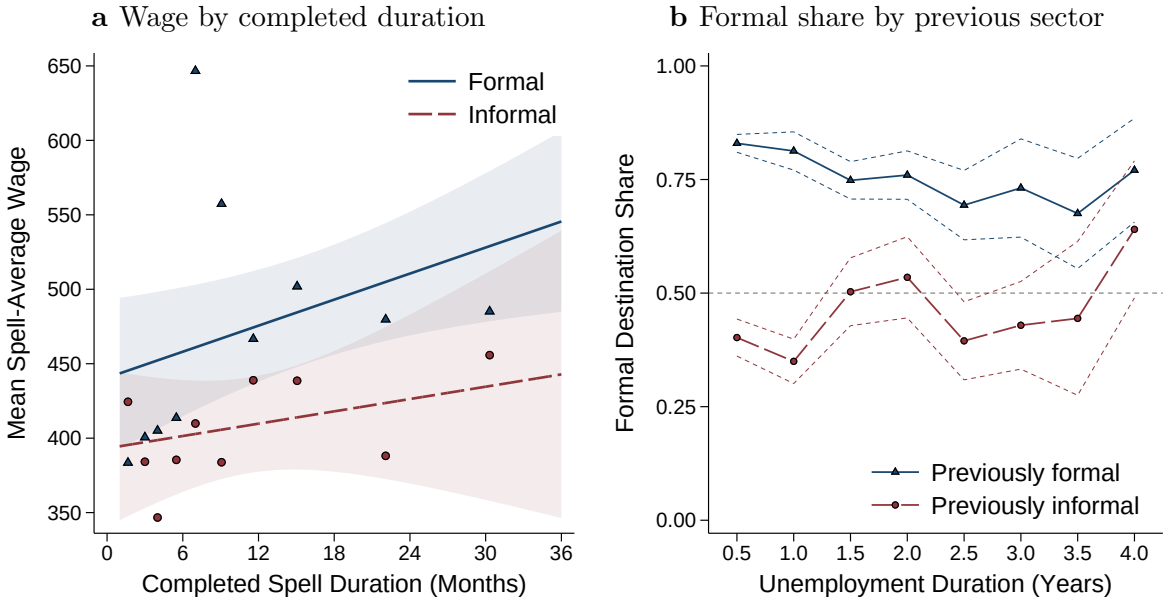
By contrast, selection in informal employment can offset the wage gains from human capital accumulation. If higher-wage informal entrants gain more from formal employment, they search more intensively and tend to leave sooner. Workers with longer completed spells can then have lower human capital at entry, making the first covariance negative. Adverse shocks can also trigger quitting, linking a spell's shock history to its duration. A relatively flat informal profile can therefore reflect human capital accumulation and selection offsetting each other.

These wage profiles leave open whether worker differences persist through unemployment. I therefore compare destinations after unemployment by previous sector in Figure 3b. In each six-month interval, the formal destination share measures the fraction of exits to employment that lead to formal jobs. This share remains higher for previously formal workers than for previously informal workers throughout the four-year horizon.

[21] This expression omits reflection at zero. When reflection occurs during a continuing employment spell, add $a_\ell \operatorname{Cov}(T_\ell, \overline{L}_{T_\ell})$, where $\overline{L}_T = T^{-1} \int_0^T (L_t - L_0)\, \mathrm{d}t$. Appendix D.1 gives the corresponding wage-average derivation.

[22] Appendix D.1 derives the wage–duration regression from Equation (D.1).

Figure 3: Wage profiles and destinations after unemployment



Notes: Panel (a) plots the flow-weighted mean wage, in real 2010 USD per month, of completed formal and informal sector-spells that begin out of unemployment and last at most 36 months (3,705 and 2,372 spells), against their duration. Bins are deciles of duration in the pooled sample, and lines are linear fits on individual spells. Panel (b) plots the flow-weighted fraction of exits from unemployment that lead to formal jobs, among exits to either employment sector. Shares are calculated by six-month interval separately for spells preceded by formal or informal employment (11,894 and 5,151 spells). Bands and dashed lines are 95% CIs clustered by worker. All samples are men ages 15–64. Source: EPS 2002–2015 rounds; author's calculations.

In the model, this gap reflects differences in the human-capital distributions of the two groups entering unemployment. At the same human capital, unemployed workers face identical search policies and human-capital dynamics, regardless of their previous sector. The destination gap therefore provides joint information about who enters and leaves each employment sector and how human capital evolves while workers are employed.

The duration comparisons, hazard profiles, and wage evidence jointly restrict explanations of access to formality. Employment spells ending in the other sector last longer than those ending in unemployment, highlighting the importance of human capital dynamics. The steeper formal wage profile further constrains how wage growth and selection interact. These patterns are consistent with the model's interaction of search costs, gains from formalization relative to the entry cost, and human capital dynamics. The observed profiles, however, do not reveal how much each mechanism contributes. Section 4 therefore estimates the model using wage and mobility moments jointly to quantify the mechanisms' individual contributions.

4 Quantitative Analysis

To measure how much each mechanism contributes, I estimate the model on the EPS employment histories in three steps. First, I set the discount rate and the formal tax wedge from external

sources. Second, I estimate the labor-force exit rate directly from the age profile. Third, I estimate the remaining 17 parameters jointly by the generalized method of moments (GMM), targeting 52 independent restrictions on labor-market allocations, wages, and the dynamics of employment spells. I then explain how these restrictions jointly discipline the parameters that shape human-capital dynamics and dynamic selection, and show how closely the estimated model reproduces them.

4.1 Moments and Estimator

External parameters. First, I set ρ to Chile's risk-free interest rate using data from its Central Bank (Banco Central de Chile, 2026). Appendix C.3 details how I construct the parameter.

Second, I use information from the OECD to recover the tax wedge τ. For each country and year, the OECD Taxing Wages database reports the income taxes and social-security contributions that workers and employers pay to the government (OECD, 2026). I use its average tax wedge for a single worker without children earning the average wage, expressed as a share of the employer's labor cost. The wedge excludes workers' contributions to their individual pension accounts, which go to private funds rather than to the government. It therefore measures exactly the revenue that enters the government's budget constraint in Equation (8). Appendix C.3 describes how I map it to the wedge described in the model.

Labor-force exit. The labor-force exit rate δ is the only parameter that I estimate directly from the EPS. In the model, workers leave the labor force at a constant rate and are replaced by entrants. In a stationary population, this constant rate gives the age distribution a convenient functional form: the log share of workers older than a given age falls linearly with age, at slope $-\delta$. I therefore recover δ directly by regressing this log share on age.[23] Appendix C.3 details the sample, weights, and regression.

GMM estimator. I estimate the remaining 17 parameters by GMM:

$$\boldsymbol{\vartheta} \equiv \left(F, b, \alpha_0, \Psi_f, \delta_i, \delta_f, \kappa_u^i, \kappa_u^f, \kappa_i^f, \kappa_f^i, \{\mu_\ell, \sigma_\ell\}_{\ell\in\{u,i,f\}}, \xi_g\right)'.$$

Three sources of variation in the EPS discipline them. The first is the allocation of workers across unemployment, informal employment, and formal employment. The second is the wages of formal and informal workers. The third is how fast and where workers move, both as their spells lengthen and in the years after they enter the labor market. Table 5 lists the moments behind each source and the parameters each moment is most informative about, and Section 4.2 develops the identification argument.

[23] The estimate absorbs both permanent attrition and actual exits from the labor force. Since the survey cannot distinguish one from the other, a rate that combines both is the consistent object for matching moments computed from the same survey.

Table 5: Parameter and GMM Mapping

Empirical family, L_g	Source	Model counterpart	Parameters
Outside of GMM			
Discount rate	Banco Central de Chile (2026)	ρ	ρ
Tax wedge	OECD (2026)	τ	τ
Age-survival profile	EPS	δ	δ
GMM moments			
Hazard (*origin*$=u$)	EPS	$\lambda_u^i(d),\ \lambda_u^f(d),\ d \in \{12, 24, 48, 54\}$	$\mu_u, \sigma_u, \kappa_u^i, \kappa_u^f, \Psi_f$
Hazard (*origin*$=i$)	EPS	$\lambda_i^f(d),\ d \in \{12, 24, 48, 60\}$	$\mu_i, \sigma_i, \kappa_i^f, \Psi_f$
Hazard (*origin*$=f$)	EPS	$\lambda_f^i(d),\ d \in \{18, 36, 54\}$	$\mu_f, \sigma_f, \kappa_f^i$
$u \to f$ share	EPS	$\chi_u^f(d),\ d \in \{18, 36, 54, 72\}$	$\mu_u, \sigma_u, \kappa_u^i, \kappa_u^f, \Psi_f$
$f \to i$ share	EPS	$\chi_f^i(d),\ d \in \{18, 36, 54, 72, 90\}$	$\mu_f, \sigma_f, \delta_f, \kappa_f^i$
$i \to f$ share	EPS	$\chi_i^f(d),\ d \in \{36, 54, 72, 90\}$	$\mu_i, \sigma_i, \delta_i, \kappa_i^f, \Psi_f$
Wages and duration	EPS	$\mathbb{E}[T_f \overline{w}_f]/\mathbb{E}[T_f],\quad \mathbb{E}[T_i \overline{w}_i]/\mathbb{E}[T_i]$	$\mu_i,\mu_f,\sigma_i,\sigma_f,\delta_i$ $\delta_f,\kappa_i^f,\kappa_f^i,\Psi_f,\alpha_0$
Labor shares	EPS	$\mathbf{M}_u, \mathbf{M}_i, \mathbf{M}_f$	b,F,α_0 $\mu_u,\sigma_u,\mu_i,\sigma_i$ $\mu_f,\sigma_f,\delta_i,\delta_f$
Entry cohort	EPS	$\mathbf{M}_u^E(d), \mathbf{M}_i^E(d), \mathbf{M}_f^E(d),\ d \in \{6, 12, \ldots, 60\}$	$\xi_g,\mu_u,\sigma_u,\kappa_u^i,\kappa_u^f$ $\mu_i,\sigma_i,\mu_f,\sigma_f,\Psi_f$ $\delta_i,\delta_f,\kappa_i^f,\kappa_f^i$

Notes: The durations targeted in the hazard and destination-share moments were selected based on the standard errors of the empirical moments.

I group the moments in Table 5 into families g, one for each profile or set of shares, and estimate $\boldsymbol{\vartheta}$ by minimizing a weighted sum of their losses:

$$\widehat{\boldsymbol{\vartheta}} = \arg\min_{\boldsymbol{\vartheta}} \left[\sum_{g \in \mathcal{G}} \omega_g L_g(\boldsymbol{\vartheta}) \right]^{1/2}, \qquad L_g(\boldsymbol{\vartheta}) = \sum_{k=1}^{K_g} c_{gk} \left[\frac{m_{gk}(\boldsymbol{\vartheta}) - \widehat{m}_{gk}}{\frac{1}{2}\left(|m_{gk}(\boldsymbol{\vartheta})| + |\widehat{m}_{gk}|\right)} \right]^2, \tag{16}$$

where ω_g is the weight on family g, and $m_{gk}(\boldsymbol{\vartheta})$ and $\widehat{m}_{gk}$ are the model and data values of its k-th moment. The weights c_{gk} sum to one within each family, so a profile does not count more because it is observed at more durations. Table 7 reports the family weights and the losses at the estimate.[24]

4.2 Identification

Before turning to how each family of moments identifies different parameters, I show that the units in which the model is measured are not identified. Every target is a share, a relative hazard, or a normalized wage. Proposition 1 shows that changing the units of time, human capital, or output rescales the parameters but leaves every target unchanged, so a parameter vector and its rescaled versions are observationally equivalent. The parameters are therefore identified only up to these units, and a normalization that fixes them selects one representative of each equivalence

[24] I combine a Sobol design with TikTak-style multistart refinement, using pattern search for local optimization (Arnoud et al., 2019).

class.

Proposition 1 (Identification up to scale)**.** *Let positive constants c_t, c_x, and c_A rescale time, human capital, and output, with every parameter rescaled according to its units. The stationary equilibrium under the rescaled parameters then generates the same targeted moments, at the same calendar durations, as the original one, so the parameters are identified only up to scale.*

Proof. See Appendix A.6.

The normalization fixes the three numbers that the targeted moments cannot identify: the length of a period, the unit of human capital, and the unit of output. I measure time in years, $a = 12$, human capital relative to the median entrant, $k_g = 1$, and output in units with $A = 1$. Because the criterion excludes the level of formal wages, I recover A from that level after estimation to express the estimates in 2010 US dollars.

I recover A by matching the mean log formal wage to its EPS counterpart. Equation (2) shows that every wage, formal or informal, is proportional to A, and so is the geometric mean formal wage. Dividing wages by that mean therefore removes A from both wage targets. I can then estimate the model without taking a stance on the units of output.

With the estimated model, I compute unobserved equilibrium outcomes such as $m_f(x)$, the distribution of human capital in the formal sector. Knowing m_f, I can split the observed mean log formal wage expressed in annual units, $\overline{\log w}_f$, into three components: the distribution of human capital, the tax wedge, and the level of output,

$$\log A = \overline{\log w}_f + \log(1+\tau) - \mathbf{M}_f^{-1} \int_0^\infty \log x \, m_f(x) \, \mathrm{d}x. \tag{17}$$

Appendix C.3 details the empirical wage scale and the computation.

Unlike A, the remaining parameters cannot be recovered one moment at a time: each family of moments depends on all of them through search policies, selection, and the stationary distribution. The arguments that follow are therefore heuristic rather than a formal proof of identification. For each family, I describe the feature of the data it records and the parameters that have first order effects on the implied moment as displayed in the last column of Table 5. These links are not exclusion restrictions, and they do not establish local or global identification.

Hazard profiles. I begin by targeting each employment hazard at longer durations relative to its value in the first six months. Recall that the hazard is the average search intensity of the workers still in the spell. Thus, when I normalize the hazard, I cancel the direct multiplicative effect of the search cost $\kappa_\ell^{\ell'}$, as shown by equation (5). Search costs can still affect these moments through the gains from moving and the composition of survivors. For unemployment, both hazards are divided by the hazard into formal employment in the first six months, so the moments discipline the ratio κ_u^f/κ_u^i.

What the moments record is how the hazards change as a spell unfolds. Theorem 2 splits that change into structural duration dependence and dynamic selection. Drift, μ_ℓ, and volatility, σ_ℓ, drive structural duration dependence, because they change survivors' human capital and, with it, their search intensity. The entry cost Ψ_f shapes dynamic selection, because it shuts off search among workers whose gain from formality falls short of Ψ_f and so widens the differences in search intensity among survivors.

Destination shares. By equation (11), a destination share divides the hazard into one destination by the sum of the hazards out of the spell. Out of unemployment, both hazards lead to employment, so the formal share is determined by the two hazards at the same duration. The share targets include durations not covered by the hazard targets in Table 5, so they constrain the model at additional durations. Out of employment, the other destination is unemployment, which a worker reaches either through an exogenous separation or by quitting at the indifference point:

$$\lambda_\ell^u(d) = \delta_\ell + \bar{q}_\ell(d), \qquad \ell \in \{i, f\}.$$

Separations and quits compete with the job-to-job hazard, which equation (5) sets as the average gain from moving divided by $\kappa_\ell^{\ell'}$. The share of exits into the other sector compares the job-to-job hazard with $\delta_\ell + \bar{q}_\ell$, so it pins down δ_ℓ relative to that hazard, that is, the product $\delta_\ell \kappa_\ell^{\ell'}$.

Wages and duration. Wages complement the mobility moments and help discipline the separation rates δ_ℓ. To see why, recall that separation rates and search costs affect how quickly spells end. In addition, human capital dynamics shape search and quitting decisions and therefore spell length. Spell length, in turn, determines how much time workers have to accumulate or lose human capital, affecting their average wages. This relationship also reflects selection, which Equation (15) in Section 3.3 captures through two terms: the covariance of duration with initial human capital and the covariance of duration with average shocks. Thus, combinations of δ_ℓ and $\kappa_\ell^{\ell'}$ that generate similar destination shares can still imply different durations and average wages. Matching wages jointly with the hazard and destination-share moments therefore helps distinguish the two parameters.

The formal–informal wage comparison also helps discipline the enforcement rate α_0. Equation (2) shows that higher enforcement lowers informal wages at a given human capital. However, the gap between the wage moments also reflects sectoral differences in human capital and spell duration. Matching wages jointly with the mobility moments helps distinguish the direct wage effect of enforcement from these compositional differences.

Labor-market shares. With α_0 disciplined by the wage gap, the labor shares pin down the benefit b and the recurring cost F. The earlier families restrict the gains from moving only relative to search costs and to one another. They leave the levels of the value functions free, and with them the thresholds at which workers favor one state over another. Because preferences are linear, b and

F shift these levels directly. Each dollar of b raises the value of unemployment by the expected discounted time a worker spends unemployed, and so draws workers into unemployment. The recurring cost F has the opposite effect on the value of formal employment, pushing workers out of it. Each movement shows up in its own share, so the unemployment share is most informative about b and the formal share about F. The informal share follows from adding up, because the three shares sum to one.

Entry-cohort trajectories. The model leaves the entrant distribution general, and for estimation I specialize it to the log-logistic family. With the median normalized to one, its cumulative distribution function is

$$G(x) = \frac{x^{\xi_g}}{1 + x^{\xi_g}}, \qquad x > 0.$$

A single parameter, ξ_g, then governs how dispersed entrants are. The mean of entrants' human capital is finite whenever $\xi_g > 1$, as Theorem 1 requires.[25]

The entry cohort disciplines the two objects that remain: ξ_g and the levels of the two unemployment search costs, whose ratio the hazard moments already fix. New workers start unemployed with human capital drawn from G, and I propagate the cohort with the model's search policies. With the other parameters anchored, the cohort's path depends on the levels of those policies. Through those levels, κ_u^i and κ_u^f set how fast the cohort leaves unemployment. The dispersion ξ_g sets how entrants are spread across search regions, and so shapes how that pace changes and how the cohort splits between sectors over its first five years.

Taken together, the families provide complementary information about the jointly estimated parameters. The hazard moments are informative about both structural duration dependence and dynamic selection, the two forces in Theorem 2, but remove the direct scale effect of search costs. The destination shares discipline separations relative to the job-to-job hazard. The wage moments add information about the level of exits, helping to distinguish δ_ℓ from $\kappa_\ell^{\ell'}$, and their gap disciplines α_0. The labor shares then anchor the levels of the value functions through b and F. Finally, the entry cohort disciplines the dispersion of entrants and the levels of the two unemployment search costs.

4.3 Estimates and Model Fit

I first present the estimates and discuss their implications for search costs, the entry cost, and human-capital accumulation in informality. Table 6 reports the estimated parameters. I then compare the model with the data family by family and trace each remaining discrepancy to a feature of the model.

The payoff parameters in Table 6 make comparative advantage follow human capital. Each unit of human capital earns 17.2 percent more in a formal job than in an informal one before taxes,

[25] The log-logistic arises as the long-run distribution of knowledge in models where agents learn from one another (Alvarez et al., 2008).

Table 6: Model Parameters

Parameter	Interpretation	Value	Parameter	Interpretation	Value
Panel A. Set outside the GMM criterion					
ρ	Discount rate (annual)	2.60%	τ	Formal tax wedge	7.53%
δ	Labor-force exit hazard (annual)	4.98%			
Panel B. Estimated by GMM					
F	Nonpecuniary cost (USD/year)	521.67	Ψ_f	Formal-entry cost (USD)	6,664.03
b	Unemployment benefit (USD/year)	0.00	ξ_g	Entrant shape and tail	7.81
α_0	Enforcement scale	14.71%	A	Production scale	10,019.37

Panel C. State-Specific Human-Capital and Mobility Parameters (GMM)

Origin state	HC drift $A\mu_\ell$ (USD/year)	Volatility $A\sigma_\ell$ (USD/$\sqrt{\text{year}}$)	Separation δ_ℓ (percent/year)	Cost to informal (USD/year)	Cost to formal (USD/year)
Unemployment	−8,774.62	0.00	—	4,743	1,708
Informal	398.15	466.98	0.10%	—	7.42
Formal	398.32	0.00	0.26%	5,525	—

Notes: Monetary values are in real 2010 USD, and the stationary employed average wage is 9,842 a year. The production scale A is in USD a year per unit of human capital. Search costs are annual flow costs at a common intensity of 0.1 offers a year. †Estimate at the boundary of the parameter space.

and 9.0 percent more after them, so the tax wedge narrows the formal advantage without closing it. The nonpecuniary cost F, by contrast, does not scale with human capital, and it outweighs this advantage when human capital is low. In the estimated equilibrium, F amounts to 49.1 percent of the formal wage that workers who favor informality would earn, and the remaining 55.6 percent of workers favor formal employment.

With the estimated search cost κ_i^f, informal workers who search for formal employment spend on average 0.27 percent of their wage on search. Yet only 5.3 percent of informal workers search, and most of those who favor formal employment do not search for it. Their gain from formality falls short of the entry cost Ψ_f, 67.7 percent of a year's average wage. This entry cost holds back essentially none of the unemployed, but 56.5 percent of informal workers.

For these held-back workers, the question is whether informality improves their prospects while they wait. Two estimates in Table 6 suggest it does. First, informal drift is positive and about equal to formal drift, so informal employment shields workers from unemployment, where human capital falls far faster than employment builds it. Second, informal jobs separate less often than formal ones, at 0.10 percent a year against 0.26, so accumulation is rarely interrupted by a return to unemployment. Informal workers, however, remain exposed to idiosyncratic shocks, which formal workers barely face. For a held-back worker, the gain from formality therefore tends to widen as the informal spell lasts, and informality becomes a place to grow into clearing the entry cost.

The estimates point to modest search spending among informal workers who search for formality, an entry cost that keeps many workers who prefer formal employment from searching, and positive informal accumulation that can help them overcome that barrier over time.

Model fit. The estimated model closely reproduces the moments that discipline these mechanisms. Table 7 summarizes this fit through the weight of each moment family in the criterion and its loss at the estimate. The criterion at the estimate is 0.17, so the typical model moment misses its data counterpart by about 17 percent. The losses show where that discrepancy comes from. The model fits the moments out of informal employment almost exactly and misses most on exits from formal employment and on the unemployment hazard into formality. Figures 4 and 5, Table 8, and Figure 6 show what lies behind these losses.

Table 7: GMM Restrictions and Fit

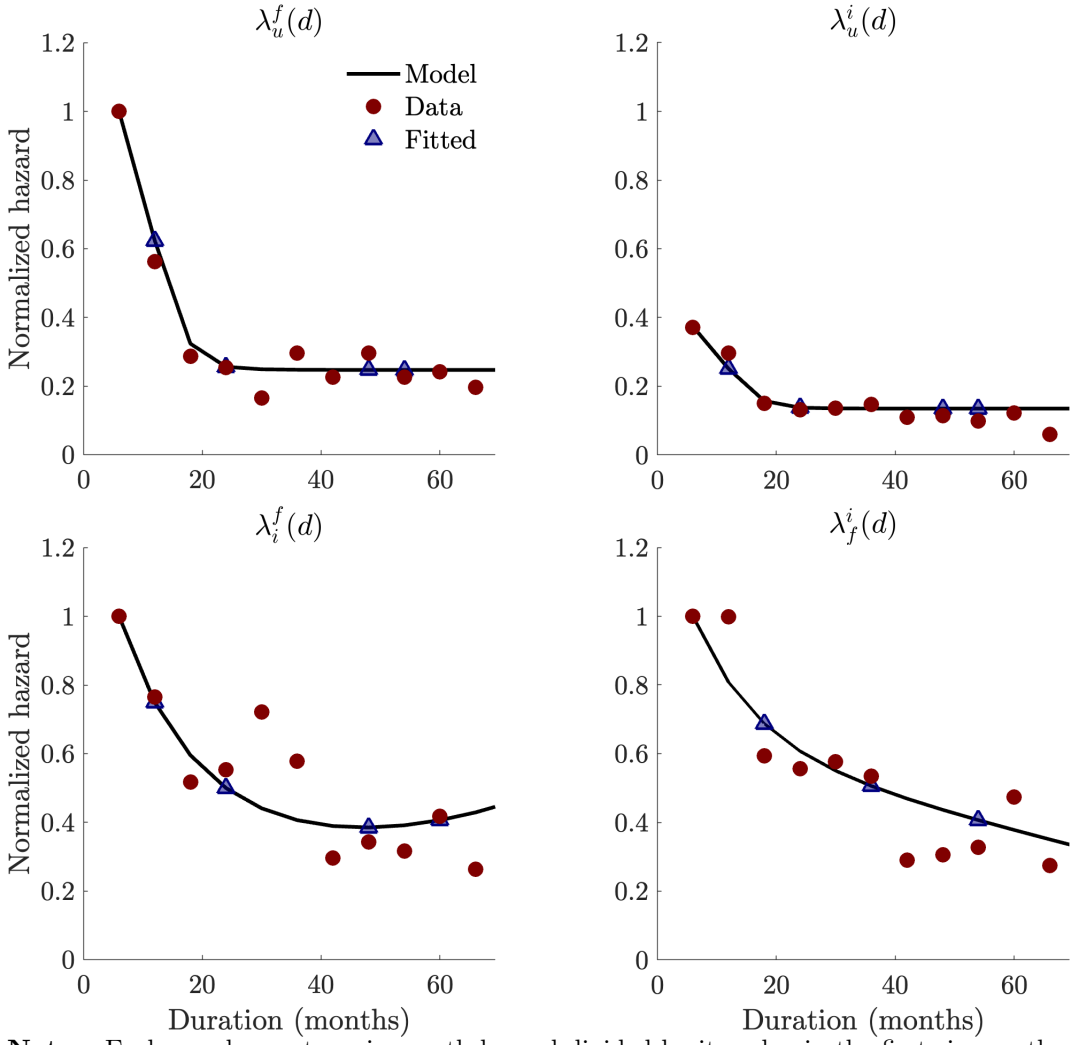
Empirical family g	n_g	Independent restrictions	ω_g	$L_g(\widehat{\boldsymbol{\vartheta}})$
Labor shares	3	2	4/33	0.02
Entry cohort	30	20	8/33	0.03
$u \to f$ share	4	4	2/33	0.00
$f \to i$ share	5	5	2/33	0.12
$i \to f$ share	4	4	2/33	0.00
Wages	2	2	2/33	0.05
$u \to f$ hazards	4	4	3/33	0.01
$u \to i$ hazards	4	4	3/33	0.04
$i \to f$ hazards	4	4	4/33	0.01
$f \to i$ hazards	3	3	3/33	0.02
Total	63	52	33/33	0.17
Jointly estimated structural parameters: $\dim(\boldsymbol{\vartheta}) = 17$				

Notes: n_g is the number of targeted objects in family g, and independent restrictions exclude adding-up identities. ω_g is the normalized family weight, and $L_g(\widehat{\boldsymbol{\vartheta}})$ is the family loss at the estimate. The total row reports the criterion in Equation (16).

Figure 4 compares the model and data hazard profiles. The estimated model attributes their shapes to structural duration dependence and dynamic selection. First, the hazards out of unemployment fall steeply in the first two years and then flatten. The steep fall reflects selection, which removes the most active searchers first. The flattening that follows reflects structural duration dependence, because human capital falls so fast in unemployment that the search incentives of those who remain vanish. Second, the hazard from formal to informal employment declines steadily, as selection removes the workers who would rather be informal. Finally, in the model as in the data, the hazard from informal to formal employment falls and then turns up after about four years. Selection again dominates the early fall. The upturn reflects accumulation in informality, which widens the gain from formality as the spell lasts. This upturn is the clearest effect of informal accumulation within the estimated model.

Where spells end tells the same story, as Figure 5 shows. Out of formal employment, selection again drives the pattern. Workers who search for informal jobs leave first, while separations into unemployment continue at a constant rate, so the informal share of exits falls with duration. The formal-origin estimates are noisy at the shortest and longest durations, where the model misses them. By contrast, the formal share of exits from informal employment inherits the U-shape of the informal-to-formal hazard, and with it the effect of accumulation.

Figure 4: Hazard Fit

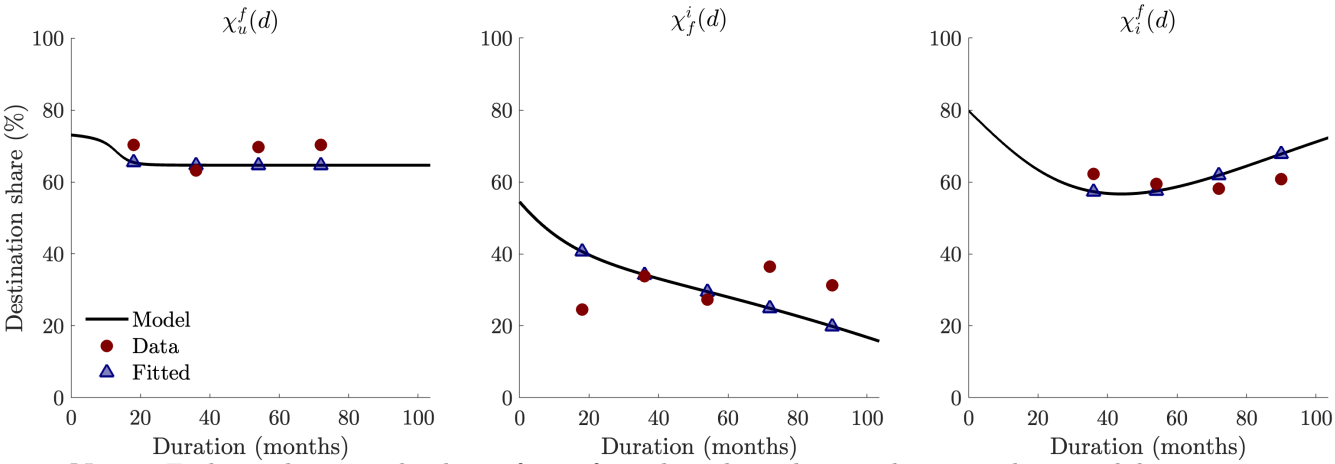


Notes: Each panel reports a six-month hazard divided by its value in the first six months, separately in the model and the data. Both unemployment hazards are divided by the hazard into formal employment. Lines are model profiles, triangles mark the model at targeted durations, and circles are EPS estimates at every six-month interval. Duration is in months.

Selection and accumulation within spells, in turn, determine the stationary allocation in Table 8, and they account for both its fit and its main miss. The model comes within 0.21 percentage points of the informal share and reproduces the formal–informal wage gap that the estimation targets. The main miss comes from the flat hazards out of unemployment. The long-term unemployed rarely leave, so the model places about 5.1 points too many workers in unemployment, mostly at the expense of formal employment. In the estimated model, long employment spells contribute to the wage overprediction because workers keep accumulating human capital. Mean wages exceed their data counterparts by 39.1 percent in formal and 54.5 percent in informal employment. Because the model matches the geometric mean formal wage through A, its higher arithmetic mean in that sector implies a larger ratio of the arithmetic to the geometric mean.

Figure 6 closes the comparison with a cohort of new workers followed from their first job search. The model reproduces the cohort's informal share throughout. It moves entrants into

Figure 5: Destination-Share Fit



Notes: Each panel reports the share of exits from the indicated origin that go to the named destination, among exits to the other two labor-market states. Lines are model profiles, triangles mark the model at targeted durations, and circles are the EPS targets.

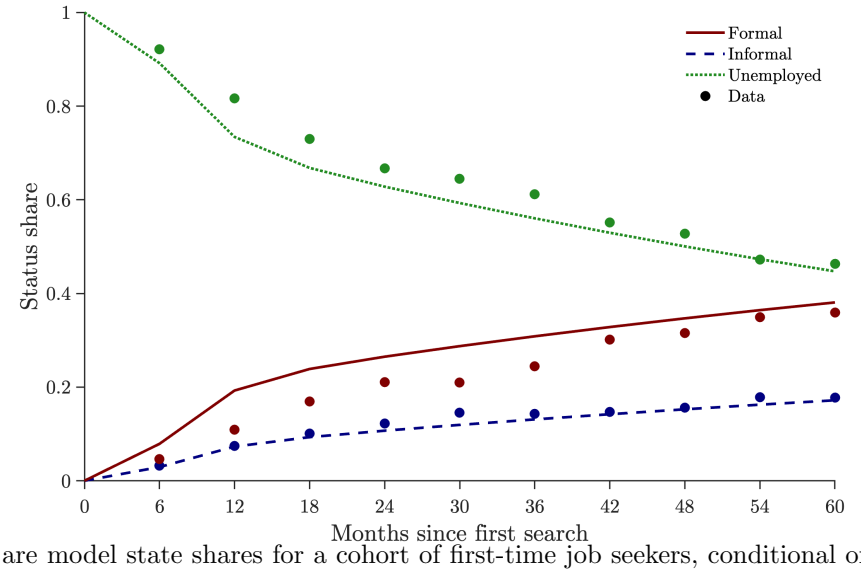
Table 8: Scalar and Wage Moment Fit

Moment	Data	Model	Difference
Panel A. Aggregate moments			
Formal employment (%)	59.83	54.47	-5.36
Informal employment (%)	20.80	21.01	+0.21
Unemployment (%)	19.37	24.52	+5.15
Panel B. Wage moments			
Formal wage mean (USD/year)	7,840	10,903	+39.1%
Informal wage mean (USD/year)	5,655	8,734	+54.5%

Notes: Panel B reports means of spell-average wages over completed spells, in real 2010 USD a year. The data use fully wage-observed, non-left-censored spells satisfying the age and occupation restrictions in Appendix B.3, with stock weights based on observed spell-month exposure. Differences are model minus data, in percentage points in Panel A and in percent in Panel B.

formal employment somewhat too quickly in the first four years, consistent with hazards out of unemployment that decline less than in the data. The gap narrows to 2.2 percentage points by the fifth year. Appendix B.4 documents the empirical cohort construction and observation rules, with the model counterpart in Appendix 1. The model therefore reproduces how new workers sort into informal and formal employment over their first years in the labor market.

Figure 6: Entry-Cohort Trajectories



Notes: Lines are model state shares for a cohort of first-time job seekers, conditional on remaining in the labor force, and circles are the EPS targets. The horizontal axis measures months since first search.

5 The Social Value of Informality

I now use the estimated model to assess how informality affects new workers' access to formal employment and their welfare. I begin by comparing the probability that the same new worker reaches formality before leaving the labor force in economies with and without informal employment. This comparison reveals a trade-off for new workers: informal employment provides a path to formality but also delays entry into the formal sector. In exchange for this delay, workers can collect wages, build human capital, and search for formal employment at a lower cost than in unemployment. However, informality dampens the gains from formalization, so workers may leave the labor force before those gains become large enough to justify the entry cost. For these reasons, informality can act as a stepping stone while still lowering the probability of a new worker reaching formality before they leave the labor force.

Despite this lower probability of formalization, informal employment has private value for new workers. Removing it therefore entails a private loss, even as the resulting expansion of formal employment increases the tax base. The additional tax revenue finances higher lump-sum transfers, creating a fiscal gain that must be weighed against the private loss. I find that the private loss outweighs this fiscal gain, so average welfare is higher when the informal option is available.

I conclude by showing that formal access can expand without the welfare loss associated with removing the informal option. To examine how, I compare greater oversight of informality, a cut in the formal tax wedge, and a subsidy to the recurring non-pecuniary cost of formal employment. Among these three policies, only the subsidy raises both output and average welfare for new workers.

5.1 Access to Formal Employment

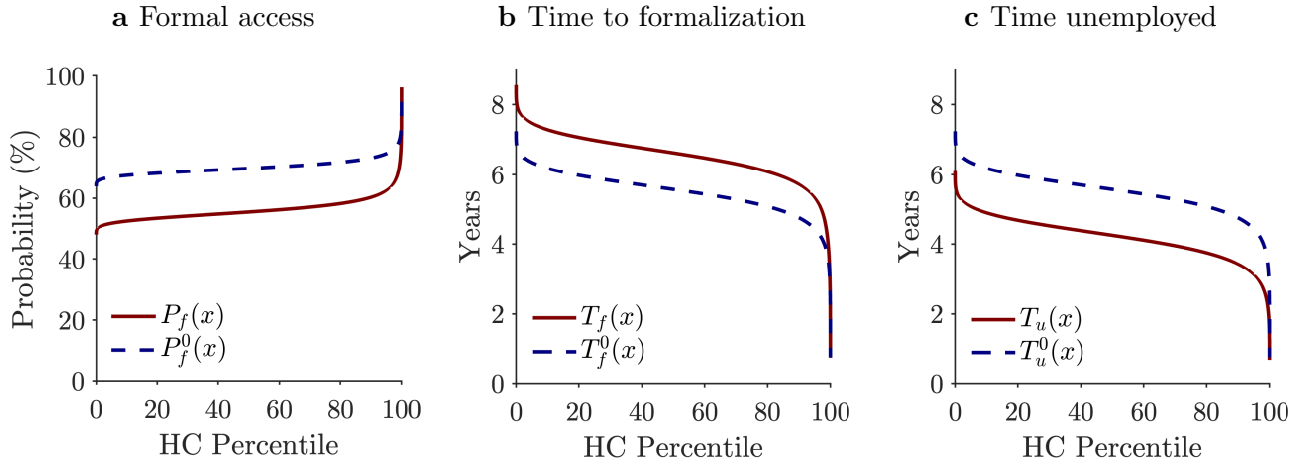
Consider an unemployed worker who just entered the labor market, with human capital x drawn from the entrant distribution G. The worker can only enter the formal sector for the first time at time T_f if they have not left the economy yet. I denote the departure moment of a worker by T_δ and the probability of the worker's eventual formalization by

$$P_f(x) = \Pr(T_f \leq T_\delta \mid x(0) = x)\,. \tag{18}$$

Consider the same worker in an economy where informal employment is no longer an option. I construct this economy by setting $\alpha_0 = 1$ and letting $\kappa_\ell^i \to \infty$ for $\ell \in \{u, f\}$. Informal wages are therefore zero, and search for informal employment is prohibitively costly. I solve for the resulting equilibrium, allowing workers to reoptimize their search and quitting decisions. Here and henceforth, I denote equilibrium objects of the economy without informality with the superscript 0.

The comparison in Figure 7a shows that formal access is higher without informality across almost the entire distribution of entrant human capital. The figure reports $P_f(x)$ for the benchmark economy and $P_f^0(x)$ for the economy without informality. Within each economy, the probability of formalization generally rises with human capital. Averaging over the entrant distribution G, I find that the probability rises from 56.10 percent in the benchmark economy to 70.18 percent without informality, an increase of 14.08 percentage points.

Figure 7: Informality and Formal Access



Notes: Panel (a) reports eventual formal-entry probabilities in percent. Panels (b) and (c) report the average time to first formalization and the average unemployment time before it, in years, conditional on formalization before labor-force exit at each initial level of human capital. Solid red lines include informality and dashed blue lines exclude it. The x-axis plots the percentiles of human capital relative to the entry distribution G.

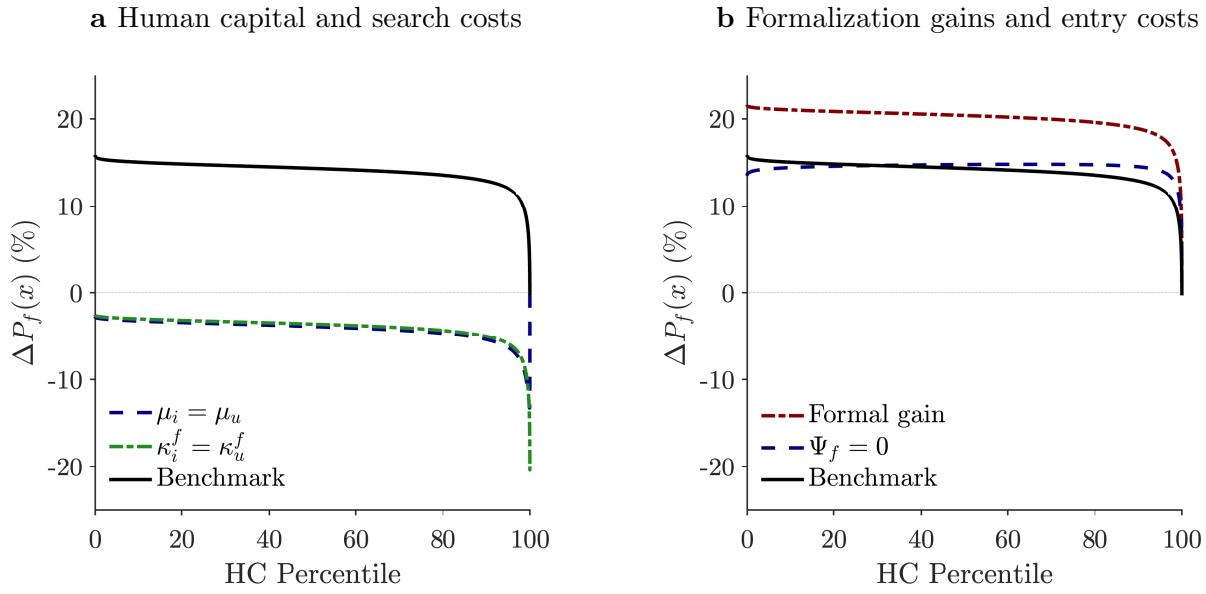
The increase in formal access comes at a cost for new workers: without an informal sector, they spend more time unemployed before becoming formal. In the benchmark economy, workers can spend time in informal employment before becoming formal, whereas without informality they remain unemployed until formal entry or labor-force exit. To quantify this trade-off, I compare the time to first formalization in Figure 7b with the unemployment time before formalization

in Figure 7c. On average, removing informality shortens the time to first formalization from 6.53 to 5.51 years but increases unemployment time before formalization from 4.19 to 5.51 years. Informality therefore delays formal entry but reduces the time workers spend unemployed before reaching formality.

Despite this delay, human capital accumulation during informal employment helps new workers reach formality. To examine the three mechanisms described in Section 2 separately, I change one mechanism at a time, holding benchmark value functions and untargeted policies fixed. I begin by setting the informal accumulation rate, μ_i, equal to its counterpart in unemployment, μ_u. I hold the benchmark search policies and quitting thresholds fixed. Figure 8a shows that this change lowers the average probability of formalization by 4.20 percentage points relative to the benchmark economy.

Lower search costs during informal employment also help new workers reach formality. To isolate this effect, I set $\kappa_i^f = \kappa_u^f$ and use equation (5) to recompute informal workers' search policy for formal employment, $\eta_i^f(x)$. I hold the gains from formalization, human capital dynamics, quitting thresholds, and all other search policies at their benchmark values. Figure 8a shows that this change lowers the average probability of formalization by 3.91 percentage points relative to the benchmark economy. Together, these two experiments show how human capital accumulation and lower search costs support formalization through informal employment.

Figure 8: Mechanisms of Formal Access



Notes: Both panels report counterfactual minus baseline eventual formal-entry probabilities, in percentage points. Colored curves use baseline value functions and hold untargeted policies fixed. The solid black benchmark reports $P_f^0(x) - P_f(x)$ in the reoptimized economy without informality. Entry-fee removal in panel (b) applies to both unemployed and informal workers. The horizontal axis uses the same entrant human-capital percentiles as Figure 7.

By contrast, informal employment weakens workers' incentives to formalize because it is more valuable than unemployment at the same human capital. For a given search cost, the smaller gain from formalization can reduce search intensity or make paying the entry cost no longer worthwhile.

To quantify this effect, I recompute informal workers' search rate as if they had the same gain from formalization as unemployed workers. Specifically, I replace $V_i(x)$ with $U(x)$ in the search rule for $\eta_i^f(x)$. I use the benchmark value functions and hold all other parameters and policies fixed. Figure 8b shows that this change raises the average probability of formalization by 20.07 percentage points relative to the benchmark economy.

Finally, the entry cost restricts formal access on both search margins: it can prevent workers from searching and reduce the intensity of those who do. To quantify its role, I set $\Psi_f = 0$ and recompute the search policies for formal employment of unemployed and informal workers, $\eta_u^f(x)$ and $\eta_i^f(x)$, using the benchmark value functions. I hold all other parameters and policies fixed and compare the resulting formalization probabilities with the benchmark to measure the entry cost's combined effect through both search margins.

The comparison shows that removing the entry cost raises average formal access by roughly as much as removing informality. Figure 8b reports an increase of 14.56 percentage points relative to the benchmark, compared with 14.08 percentage points from removing the informal option. For workers with low initial human capital, however, removing the entry cost does not fully close the access gap with the economy without informality. These workers still face the recurring cost of formal employment, F, and may need to accumulate human capital in informal employment before formalization becomes worthwhile.

5.2 Welfare and Fiscal Gains

I next examine the welfare trade-off created by removing informality: workers lose a valuable employment option but benefit from higher lump-sum transfers. The private loss reflects the value of earning informal wages while building human capital and searching for formal employment at a lower cost than in unemployment. On the fiscal side, removing informality raises the share of formal workers from 54.47 to 69.12 percent, expanding the tax base and government revenue. The additional revenue finances higher lump-sum transfers, creating a fiscal gain that must be weighed against the private loss.

To compare the private loss with the fiscal gain, I measure a new worker's welfare as the sum of private value and discounted lump-sum transfers. All workers enter the labor market unemployed, so welfare for an entrant with human capital x is

$$W(x) = U(x) + \frac{\mathcal{T}}{\rho + \delta}. \tag{19}$$

Here, $U(x)$ is the value of entering unemployed, excluding common transfers, and $\mathcal{T}/(\rho+\delta)$ is the present discounted value of receiving those transfers before labor-force exit. As before, superscript 0 denotes equilibrium objects in the economy without informality.

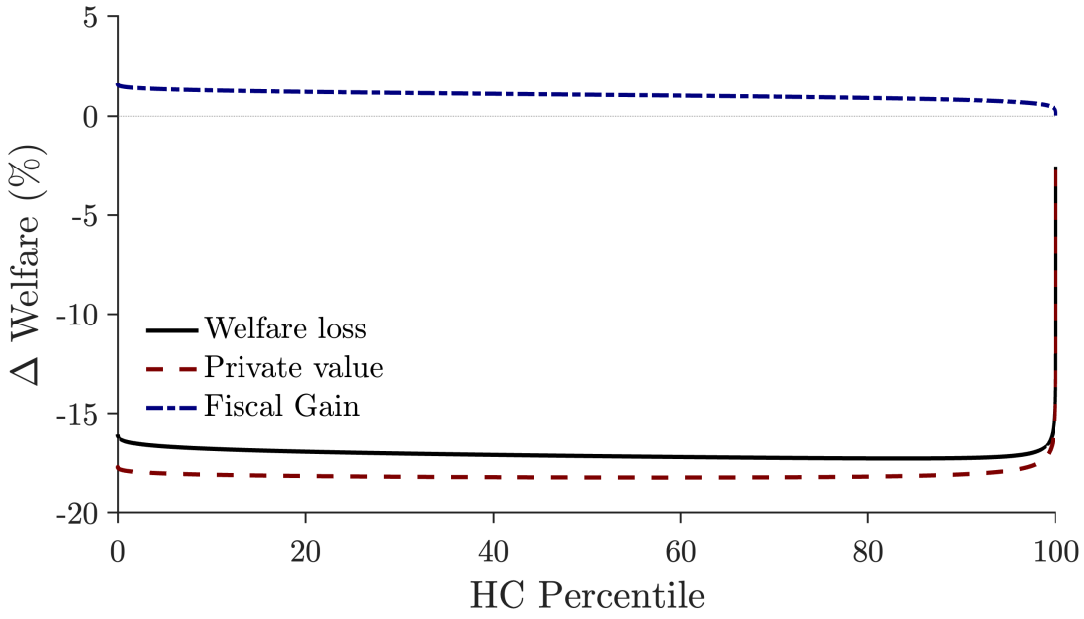
I use equation (19) to decompose the welfare effect of removing informality into a private loss

and a fiscal gain,

$$\Delta W(x) = \Delta U(x) + \frac{\Delta \mathcal{T}}{\rho + \delta}.$$

Here, $\Delta W(x) = W^0(x) - W(x)$ is the welfare change from removing informality, with $\Delta U(x)$ and $\Delta \mathcal{T}$ defined in the same way. The private component, $\Delta U(x)$, captures the loss from removing the informal option, while the transfer component, $\Delta \mathcal{T}/(\rho + \delta)$, captures the gain from higher lump-sum transfers. Figure 9 plots the two components and their sum across percentiles of entrant human capital. Each curve is expressed as a percentage of benchmark welfare, $W(x)$. Figure 9

Figure 9: Welfare Losses from Removing Informality



Notes: All three curves are expressed as percentages of initial welfare $W(x)$ in the economy with informality. Differences are without informality minus with informality, so welfare and private-value losses are negative and fiscal gains are positive. The fiscal component is the discounted common-transfer difference $\Delta \mathcal{T}/(\rho+\delta)$ in Equation (19). The horizontal axis uses entrant human-capital percentiles.

shows that the private loss from removing informality outweighs the fiscal gain. The expansion of the tax base raises government revenue by 7.79 percent, but the resulting fiscal gain averages only 1.06 percent of benchmark welfare. By contrast, the private component averages -18.09 percent, yielding a net average welfare change of -17.02 percent.

Taking stock, informality raises average welfare for new workers even though it delays formal entry and lowers the probability of reaching formality. Informal employment allows workers to earn wages, build human capital, and search for formal employment at a lower cost than in unemployment. Removing this alternative expands the tax base, but the resulting fiscal gain is too small to compensate for the private loss. I therefore examine whether formal access can be expanded without lowering average welfare for new workers.

5.3 Formalization Policies

Policies can promote formalization by making informal employment less attractive or formal employment more attractive. To compare the private and fiscal consequences of these approaches, I consider three instruments. First, I study a stick policy that increases oversight of informality, α_0, and lowers informal wages. Second, I consider a carrot policy that reduces the tax wedge, τ, and raises the wages received by formal workers. Finally, I examine a government subsidy that partly offsets the recurring non-pecuniary cost of formal employment, F.

To implement the subsidy, the government offsets a share s of the recurring cost F by paying sF to each formal worker.[26] The government budget becomes

$$\tau \int_0^\infty w_f(x) m_f(x)\, \mathrm{d}x = b\, \mathbf{M}_u + \mathcal{T} + sF\mathbf{M}_f,$$

where $\mathcal{T}$ is the common lump-sum transfer received by all workers. The payment reduces the effective burden of formal employment to $(1-s)F$. In Table 9, I report the subsidy's financing requirement as the share of tax revenue net of unemployment benefits allocated to these payments.

To compare the welfare effects of the three policies, I calibrate each to deliver a common increase p in new workers' average probability of formalization. Let $P_f^r(x)$ denote the probability of formalization for a new worker with human capital x under policy $r \in \{\alpha_0, \tau, s\}$. I choose each instrument's value to satisfy

$$\int_0^\infty \left[P_f^r(x) - P_f(x)\right] g(x)\, \mathrm{d}x = p, \tag{20}$$

with a target of 5 percentage points.[27] Table 9 reports the calibrated policy values and their outcomes.

First, consider increasing oversight of informality, α_0: this policy produces a small welfare gain but reduces output. To deliver the common increase of 5 percentage points in formal access, the government raises α_0 to 18.37%, 3.66 percentage points above its benchmark value. By lowering informal wages, this increase encourages formalization and expands the tax base, raising tax revenue by 10.43 percent. At the same time, the private welfare effect of these lower wages is -1.11 percent of mean baseline entrant welfare. Nevertheless, the increase in lump-sum transfers outweighs this private loss, yielding a net welfare gain of 0.26 percent. However, Table 9 reports an output change of -0.41 percent before enforcement losses. Greater oversight also increases the fraction of informal production lost to enforcement.

Unlike greater oversight of informality, the tax cut I consider next raises output but lowers average welfare. To deliver the common increase in formal access, the government cuts τ by 3.56

[26] This last policy tries to capture the argument made by Levy (2008), in which he claims that policy should try to level the playing field for formal and informal workers.

[27] Reducing the effective tax wedge through contribution matching produces a similarly sized increase in pension enrollment (Bernal et al., 2025).

Table 9: Formalization Policies at a Common Access Gain

	Higher α_0	Lower τ	F subsidy
Policy value	18.37%	3.97%	38.43%
	(3.66)	(3.56)	(62.11)
Private gain (%)	−1.11	4.23	4.59
Δ Tax Revenue (%)	10.43	−39.54	9.79
Welfare gain (%)	0.26	−0.94	0.35
ΔY (%)	−0.41	0.99	0.82

Notes: All policies raise formal access by 5 pp in Equation (20). Parentheses report magnitudes of policy changes in pp for the first two columns and the percentage reduction in the effective formal burden for the subsidy. The subsidy policy value is its share of post-benefit tax revenue. Private and welfare gains are percentages of mean baseline entrant welfare. Output Y is measured before enforcement losses.

percentage points to 3.97 percent. By increasing the wages received by formal workers, this policy makes formal employment more attractive and generates a private gain equivalent to 4.23 percent of baseline welfare. However, the expansion of the tax base is not enough to compensate for the lower tax rate, and tax revenue falls by 39.54 percent. The resulting decline in lump-sum transfers outweighs the private gain, yielding a net welfare loss of 0.94 percent. Output nevertheless increases by 0.99 percent.

Finally, I find that subsidizing the recurring non-pecuniary cost of formal employment, F, raises both output and average welfare. To deliver the same increase in formalization, the government must allocate 38.43 percent of tax revenue net of unemployment benefits to the subsidy, offsetting 62.11 percent of F. This payment generates a private gain equivalent to 4.59 percent of baseline welfare. The expansion of the tax base also increases tax revenue by 9.79 percent. Despite these additional receipts, financing the subsidy requires reducing the lump-sum transfers received by all workers. This fiscal loss absorbs most of the private gain, leaving a net welfare increase of 0.35 percent, while output also increases by 0.82 percent.

Taken together, these experiments show that, at the common formalization target, only the subsidy raises both output and average welfare for new workers. Greater oversight raises tax revenue and welfare but lowers output even before accounting for enforcement losses. Cutting the tax wedge instead raises output and private welfare, but the resulting revenue loss reduces lump-sum transfers enough to lower average welfare. Translating the subsidy result into policy, however, requires identifying the recurring burdens represented by F that government spending can alleviate. Commuting costs provide one possible interpretation of this burden. For example, Zárate (2026) shows how transit improvements in Mexico City reduce commuting frictions and expand access to formal employment. These findings therefore point to easing the recurring burdens of formal employment as a promising way to expand formal access while raising output and improving welfare for new workers.

6 Conclusion

To conclude, informal employment provides a stepping stone to formality while lengthening the path and lowering the probability of reaching it. In the estimated model, workers entering the labor market can earn income, build human capital, and search for formal employment at lower cost than in unemployment. Yet the value of informal employment reduces the gain from formalizing, weakening incentives to search intensively and incur entry costs. Workers may therefore leave the labor force before the gains from formalization justify the entry cost. Although I apply this framework to informality, it can also be used to study how human capital dynamics and search incentives shape transitions in other settings, such as the gig economy.

A lower probability of reaching formal employment does not imply, however, that the informal option lowers workers' welfare. Removing informality raises formal access and expands the tax base, but lowers average welfare for workers entering the labor market. Workers lose the opportunity to earn informal wages while building human capital and searching for formal employment at lower cost. The broader tax base finances higher common transfers, but this fiscal gain is too small to compensate for the private loss. Formal access alone therefore misses the value workers derive from informal employment along the way.

Effective formalization policy needs to account for the private value of informality. The welfare consequences of expanding formal access depend on which costs policy targets and how the reform is financed. Among the three policies compared at a common formal-access gain, only the subsidy to the recurring cost of formal employment raises both output and average entrant welfare. Greater oversight lowers informal wages, whereas a tax cut raises formal wages but reduces revenue and common transfers. The subsidy makes formal employment more attractive while retaining the informal option. Financing it also reduces common transfers, but the private gain exceeds the fiscal loss. These results point to easing the recurring burdens of formal employment as a way to expand access while preserving the benefits of informal employment. Consequently, implementation requires identifying which burdens represented by F public spending can alleviate.

References

Alvarez, F., Borovičková, K., & Shimer, R. (2024). Decomposing Duration Dependence in a Stopping Time Model. *The Review of Economic Studies*, *91*(6), 3151–3189.

Alvarez, F., Buera, F. J., & Lucas, R. E., Jr. (2008). *Models of idea flows* (Working Paper No. 14135). Cambridge, MA: National Bureau of Economic Research.

Alvarez, F., & Shimer, R. (2011). Search and rest unemployment. *Econometrica*, *79*(1), 75–122.

Arnoud, A., Guvenen, F., & Kleineberg, T. (2019). *Benchmarking Global Optimizers* (NBER Working Paper No. 26340). National Bureau of Economic Research.

Banco Central de Chile. (2026). *Tasa de política monetaria (TPM) y tasas de bonos licitados por el Banco Central de Chile.* Base de Datos Estadísticos.

Bernal, N., Galiani, S., & Molina, O. (2025). *Does matching contribution incentivize informal workers to participate in retirement saving plans? a randomized evaluation interacted with a natural experiment* (NBER Working Paper No. 33925). National Bureau of Economic Research.

Bobba, M., Flabbi, L., & Levy, S. (2022). Labor Market Search, Informality, and Schooling Investments. *International Economic Review*, *63*(1), 211–259.

Bobba, M., Flabbi, L., Levy, S., & Tejada, M. (2021). Labor Market Search, Informality, and On-the-Job Human Capital Accumulation. *Journal of Econometrics*, *223*(2), 433–453.

Bobba, M., Gadenne, L., Harari, M., & Ulyssea, G. (2025). Informality. *VoxDevLit*, *6*(2). (Version dated April 24, 2025)

Bosch, M., & Maloney, W. F. (2010). Comparative Analysis of Labor Market Dynamics using Markov Processes: An Application to Informality. *Labour Economics*, *17*(4), 621–631.

Cano-Urbina, J. (2015). The Role of the Informal Sector in the Early Careers of Less-Educated Workers. *Journal of Development Economics*, *112*, 33–55.

Castro, R., Lange, F., & Poschke, M. (2024). *Labor Force Transitions* (Working Paper No. 33200). National Bureau of Economic Research.

Chile. (2003). *Código del trabajo, decreto con fuerza de ley núm. 1, artículo 9.* Biblioteca del Congreso Nacional de Chile, Ley Chile. Retrieved 2026-08-04. (Promulgated July 31, 2002; published January 16, 2003)

Crandall, M. G., Ishii, H., & Lions, P.-L. (1992). User's Guide to Viscosity Solutions of Second Order Partial Differential Equations. *Bulletin of the American Mathematical Society*, *27*(1), 1–67.

de Mel, S., McKenzie, D., & Woodruff, C. (2013). The demand for, and consequences of, formalization among informal firms in Sri Lanka. *American Economic Journal: Applied Economics*, *5*(2), 122–150.

Donovan, K., Lu, W. J., & Schoellman, T. (2023). Labor Market Dynamics and Development. *The Quarterly Journal of Economics*, *138*(4), 2287–2325.

Finamor, L. (2025). *Labor Market Informality, Risk, and Insurance.* (Working paper)

Franco, S., & Quintero, J. M. (2026). *Learning Segmentation and Economic Development.* (Working paper; substantially revised version of (In)formal Growth)

La Porta, R., & Shleifer, A. (2014). Informality and Development. *Journal of Economic Perspectives*, *28*(3), 109–126.

Levy, S. (2008). *Good intentions, bad outcomes: Social policy, informality, and economic growth in mexico.* Washington, DC: Brookings Institution Press.

Lucas, R. E., Jr., & Prescott, E. C. (1974). Equilibrium search and unemployment. *Journal of Economic Theory*, *7*(2), 188–209.

López García, I. (2015). *Human Capital and Labor Informality in Chile: A Life-Cycle Approach* (Working Paper No. WR-1087). RAND Corporation.

Magnac, T. (1991). Segmented or Competitive Labor Markets. *Econometrica*, *59*(1), 165–187.

Meghir, C., Narita, R., & Robin, J.-M. (2015). Wages and Informality in Developing Countries. *American Economic Review*, *105*(4), 1509–1546.

OECD. (2026). *Labour taxation—OECD member country tables.* OECD Data Explorer. (Official OECD member-country dataflow, version 2.1; the average tax wedge is reported as a percentage of labour costs)

Olken, B. A., Hanna, R., Poonpolkul, P., & Wasi, N. (2024). *Willingness-to-pay vs administrative hurdles: Understanding barriers to social insurance enrollment in Thailand* (NBER Working Paper No. 33096). National Bureau of Economic Research.

Perry, G. E., Maloney, W. F., Arias, O. S., Fajnzylber, P., Mason, A. D., & Saavedra-Chanduvi, J. (2007). *Informality: Exit and Exclusion.* Washington, DC: World Bank.

Seminario-Amez, C. (2021). *Informal Rungs on the Job Ladder: Theory and Evidence from Brazil.* (Job Market Paper)

Ulyssea, G. (2018). Firms, Informality, and Development: Theory and Evidence from Brazil. *American Economic Review*, *108*(8), 2015–2047.

Zárate, R. D. (2026). *Spatial misallocation, informality, and transit improvements: Evidence from Mexico City.* (Working paper, UC San Diego)

Appendix Contents

A Theoretical Appendix **A-1**

A.1 Worker Problem and Optimal Policies A-1
A.2 Worker Distributions and Population Accounting A-10
A.3 Stationary Equilibrium A-16
A.4 Sorting and Duration Dependence A-22
A.5 Hazard Estimation A-25
A.6 Identification up to Scale A-27

B Data Appendix **B-1**

B.1 EPS Sources and Sample Construction B-1
B.2 Labor-Market States and Sector-Spells B-5
B.3 Wage Measurement B-7
B.4 First-Time Job-Search Cohort B-8

C Estimation and Numerical Methods **C-1**

C.1 Solving the Stationary Equilibrium C-1
C.2 Numerical Calculation of Model Moments C-6
C.3 External Calibration and Normalizations C-10

D Additional Results **D-1**

D.1 Additional Descriptive Evidence D-1
D.2 Robustness of the Empirical Evidence D-4
D.3 Additional Model Results D-5

A Theoretical Appendix

A.1 Worker Problem and Optimal Policies

This subsection establishes the properties of the stationary worker values that shape workers' comparative advantage and search decisions. The values grow at most linearly and are nondecreasing, Lipschitz, and convex in human capital. These properties then determine where workers quit and how values and search behave at high human capital.

Wages are stationary, $w_\ell(x) = a_\ell x$ with the coefficients in equation (2), so I suppress calendar time. Both coefficients are positive, since $\alpha_0 < 1$, and I write $\bar{a} = \max\{a_i, a_f\}$. Results that need a formal wage advantage, $a_f > a_i$, state it.

The proofs compare each value with the payoff of a particular plan. A plan chooses search intensities and quitting dates from the information available to date.[28] Write $W_\ell(x;\pi)$ for the expected lifetime payoff of an admissible plan π in state $\ell \in \{u, i, f\}$ at human capital x, and $W_\ell(x)$ for its supremum, so that $W_u = U$, $W_i = V_i$, and $W_f = V_f$. Let τ_δ be the date of labor-force exit.

Bounds and shape. The first result supplies the bounds that the later comparisons use. Intuitively, a worker can always secure the benefit by remaining unemployed, while no plan earns more than the highest wage slope applied to accumulated human capital. Values therefore lie between the value of permanent unemployment, $b/(\rho+\delta)$, and a linear upper bound. Its slope, $\bar{a}/(\rho+\delta)$, is the value of earning $\bar{a}$ on a unit of human capital until labor-force exit.

Lemma 1 (Value bounds and transversality)**.** *The stationary worker values have the following properties.*

(a) There is a finite constant B, independent of search costs, such that for every $\ell \in \{u, i, f\}$,

$$\frac{b}{\rho+\delta} \le W_\ell(x) \le \frac{\bar{a}}{\rho+\delta}x + B, \qquad x \ge 0. \tag{A.1}$$

(b) The formal value satisfies

$$V_f(x) \ge \frac{b}{\rho+\delta} + \frac{a_f x - F - b}{\rho+\delta+\delta_f}. \tag{A.2}$$

(c) For every worker value W_ℓ and every admissible plan, $\mathbb{E}_x[e^{-\rho T}|W_\ell(x(T))|\mathbb{1}\{\tau_\delta > T\}] \to 0$ as $T \to \infty$. The same limit holds when survival is restricted to the current spell.

[28] An admissible plan has nonnegative, predictable search intensities and quitting dates that are stopping times. Cumulative intensities are finite on finite horizons, transitions are nonexplosive, and discounted payoffs and costs are integrable. The Brownian motion, search-arrival drivers, separation clocks, and exit clock are mutually independent on a complete, right-continuous filtered probability space.

Proof. Fix an initial state ℓ, initial human capital x, and an admissible plan π. I first show that expected human capital rises at most linearly in time and that the bound is uniform over admissible plans. Integrating this estimate against the discount factor $e^{-(\rho+\delta)s}$ gives the upper bound in (a), and evaluating it at a date T gives (c). The lower bounds in (a) and (b) come from two simple plans.

To separate human-capital dynamics from survival, evaluate the plan along histories without labor-force exit. Survival then enters payoffs only through the effective discount rate $\rho + \delta$. Let $j(s)$ be the pretransition labor-market state in this exit-free process. The process is independent of τ_δ and agrees with the worker's state on $\{\tau_\delta > s\}$. Admissibility gives nonexplosion and a predictable pretransition state, and human capital is continuous across transitions. Define

$$z(s) = \int_0^s \mu_{j(r)}\,\mathrm{d}r + M(s), \qquad M(s) = \int_0^s \sigma_{j(r)}\,\mathrm{d}W_r, \qquad k(s) = -\inf_{0\le r\le s} z(r).$$

The one-dimensional Skorokhod representation of the reflected path then gives $x(s) = z(s) + \max\{x, k(s)\}$.

Write $\bar\mu = \max_j |\mu_j|$ and $\bar\sigma = \max_j \sigma_j$, where $j \in \{u, i, f\}$. The maximum in this representation is at most $x + \bar\mu s + \sup_{r\le s} |M(r)|$, and the drift part of $z(s)$ is at most $\bar\mu s$. Doob's inequality and the bound $\mathbb{E}[M(s)^2] \le \bar\sigma^2 s$ imply $\mathbb{E}[\sup_{r\le s} |M(r)|] \le 2\bar\sigma\sqrt{s}$. Since $\mathbb{E}[M(s)] = 0$, taking expectations in the reflected-path representation gives, uniformly over admissible plans,

$$\mathbb{E}_x[x(s)] \le x + 2\bar\mu s + 2\bar\sigma\sqrt{s}. \tag{A.3}$$

Upper bound in (a). Search, formal-employment, and transition costs are nonnegative, so flow payoffs are at most $b + \bar a x(s)$. Because labor-force exit is independent and occurs at rate δ, discounting at ρ until exit is equivalent to discounting at $\rho + \delta$. With equation (A.3), this gives

$$\begin{aligned} W_\ell(x;\pi) &\le \int_0^\infty e^{-(\rho+\delta)s}\big[b + \bar a\,\mathbb{E}_x[x(s)]\big]\,\mathrm{d}s \\ &\le \int_0^\infty e^{-(\rho+\delta)s}\big[b + \bar a(x + 2\bar\mu s + 2\bar\sigma\sqrt{s})\big]\,\mathrm{d}s = \frac{\bar a}{\rho+\delta}x + B. \end{aligned}$$

Since $\rho + \delta > 0$, the constant B is finite. It depends only on benefits, wage and human-capital coefficients, and $\rho + \delta$, so it is the same for every initial state and plan and does not depend on search costs. Taking suprema over plans gives the upper bound for each value.

Lower bounds in (a) and (b). Each lower bound comes from a simple feasible plan. An unemployed worker who never searches receives b until labor-force exit, which is worth $b/(\rho + \delta)$. An employed worker can quit immediately and then follow the same plan. Taking suprema gives the lower bound in (a). For (b), consider a formal worker who never searches, keeps the job until exogenous separation, and then remains unemployed without searching. Since $\mu_f > 0$ and reflection only adds human capital, expected human capital during the formal spell is at least x. Discounting the formal spell at $\rho+\delta+\delta_f$ and the subsequent unemployment at $\rho+\delta$ gives equation

(A.2).

Transversality. Part (a) provides a finite C with $|W_\ell(x)| \le C(1+x)$ for every ℓ. Independence of exit and equation (A.3) then give

$$\begin{aligned}\mathbb{E}_x\left[e^{-\rho T}\left|W_\ell(x(T))\right|\mathbb{1}\{\tau_\delta > T\}\right] &= e^{-(\rho+\delta)T}\mathbb{E}_x[|W_\ell(x(T))|] \\ &\le Ce^{-(\rho+\delta)T}\left(1 + x + 2\bar{\mu}T + 2\bar{\sigma}\sqrt{T}\right) \longrightarrow 0.\end{aligned}$$

Restricting survival to the current spell only reduces this nonnegative term, so the same limit holds. □

Monotonicity, Lipschitz continuity, and convexity all rest on one construction. Note that a worker can imitate the search and quitting decisions of a worker who starts with different human capital. The two workers then follow the same labor-market path and pay the same costs, so their payoffs differ only through wages.

Proposition 2 (Monotonicity of worker values)**.** *Each worker value is nondecreasing in human capital: $W_\ell(y) \ge W_\ell(x)$ for every $\ell \in \{u, i, f\}$ and $y \ge x \ge 0$.*

Proof. Fix an initial state ℓ, levels of human capital $y \ge x$, and an admissible plan π at x. It suffices to construct an admissible plan at y whose payoff is at least $W_\ell(x;\pi)$, because taking the supremum over π then gives $W_\ell(y) \ge W_\ell(x)$. The plan at y imitates π. I first check that this imitation is admissible, then compare the two payoffs.

Imitation. A worker starting from y can use x and the primitive shocks observed to date to reconstruct the history that π would have generated. The worker then follows the search intensities and quitting decisions prescribed by π. On histories outside this construction, the plan specifies zero search and immediate quitting if employed. Copied intensities are predictable, and copied quitting times are stopping times. Because the two workers share the arrival drivers and separation clocks and take the same actions, their next transition is the same. By induction and nonexplosion, the labor-market paths coincide on every finite horizon. Search costs, entry costs, and transition dates coincide, so cost integrability is inherited from π, and equation (A.3) gives integrability of flow income. Hence the imitating plan is admissible.

Payoff comparison. Let $j(s)$ be the common labor-market path, and define $z(s)$ and $k(s)$ along it as in the proof of Lemma 1. The two reflected paths are $x(s;x) = z(s) + \max\{x, k(s)\}$ and $x(s;y) = z(s) + \max\{y, k(s)\}$, so

$$0 \le x(s;y) - x(s;x) \le y - x. \tag{A.4}$$

The worker starting from y thus holds weakly more human capital at every date and earns weakly higher wages, while benefits and costs coincide. Consequently, the imitating plan yields at least $W_\ell(x;\pi)$, which is what was needed. □

Lemma 2 (Global Lipschitz continuity and convexity)**.** *For every $\ell \in \{u, i, f\}$, the value W_ℓ is convex, and*

$$0 \le W_\ell(y) - W_\ell(x) \le \frac{\bar{a}}{\rho + \delta}(y - x), \qquad y \ge x \ge 0.$$

In particular, W_ℓ has one-sided derivatives on $(0, \infty)$, with $0 \le \partial_x^- W_\ell(x) \le \partial_x^+ W_\ell(x) \le \bar{a}/(\rho + \delta)$.

Observe that the Lipschitz constant is the slope of the upper bound in Lemma 1. It bounds the gain from an extra unit of human capital by the value of earning $\bar{a}$ until labor-force exit.

Proof. Both properties follow from the imitation in the proof of Proposition 2. An imitating plan reproduces the labor-market path and all costs, so payoffs differ only through wages, and wages differ only through human capital. For the Lipschitz bound, a worker with less human capital imitates one with more, and equation (A.4) limits the wage gap. For convexity, workers at two endpoints imitate a worker at an intermediate point. Convexity of the reflected path in initial human capital gives the required payoff comparison because wages are affine in human capital with nonnegative slopes and all costs coincide.

Lipschitz continuity. Fix ℓ, $y \ge x$, and an admissible plan π_y at y. The imitation yields an admissible plan $\tilde{\pi}_x$ at x with the same labor-market path and costs as π_y, and equation (A.4) keeps the human-capital gap in $[0, y - x]$. Since wage slopes, including $a_u = 0$, lie in $[0, \bar{a}]$,

$$W_\ell(y; \pi_y) - W_\ell(x; \tilde{\pi}_x) \le \bar{a}(y - x)\mathbb{E}\left[\int_0^{\tau_\delta} e^{-\rho s}\, \mathrm{d}s\right] = \frac{\bar{a}}{\rho + \delta}(y - x).$$

Replacing $W_\ell(x; \tilde{\pi}_x)$ by the larger $W_\ell(x)$ and taking the supremum over π_y gives the upper bound. Proposition 2 gives the lower bound.

Convexity. Fix $x_0, x_1 \ge 0$, $\lambda \in [0, 1]$, and $x_\lambda = \lambda x_1 + (1 - \lambda)x_0$. For an admissible plan π_λ at x_λ, workers starting at x_0 and x_1 imitate it. This gives admissible plans $\tilde{\pi}_0$ and $\tilde{\pi}_1$ with the same labor-market path and costs. Along this path, $z(s)$ and $k(s)$ are common, and $x \mapsto z(s) + \max\{x, k(s)\}$ is convex, so $x(s; x_\lambda) \le \lambda x(s; x_1) + (1 - \lambda)x(s; x_0)$ for every s. Flow income in each state is affine in human capital with a nonnegative slope, and all costs are common. Integrating along paths and taking expectations therefore gives

$$\begin{aligned} W_\ell(x_\lambda; \pi_\lambda) &\le \lambda W_\ell(x_1; \tilde{\pi}_1) + (1 - \lambda)W_\ell(x_0; \tilde{\pi}_0) \\ &\le \lambda W_\ell(x_1) + (1 - \lambda)W_\ell(x_0). \end{aligned}$$

Taking suprema over π_λ proves convexity. Finally, a convex function whose increments lie in $[0, \bar{a}(y - x)/(\rho + \delta)]$ has one-sided derivatives in $[0, \bar{a}/(\rho + \delta)]$, ordered as stated. □

Quitting. For $\ell \in \{i, f\}$, the employment surplus $D_\ell = V_\ell - U$ is the gain from holding a job in ℓ rather than being unemployed. Because workers can quit, $D_\ell \ge 0$, and the quitting and

continuation sets are

$$\mathcal{S}_\ell = \{x \geq 0 : D_\ell(x) = 0\}, \qquad \mathcal{C}_\ell = \{x \geq 0 : D_\ell(x) > 0\}.$$

Whether quitting is confined to low human capital depends on how the surplus changes with human capital. A higher level of human capital raises the value of a job in ℓ through the wage and through human-capital accumulation, which is never slower than in unemployment. The value of unemployment rises as well, because unemployed workers search for jobs whose wages increase with human capital. When the gain in the job value is at least as large as the gain in the unemployment value, D_ℓ is nondecreasing in human capital. I call this property increasing differences between employment and unemployment and maintain it where stated.[29] Under increasing differences, unemployment search $\eta_u^\ell = [D_\ell - \Psi_\ell]^+/\kappa_u^\ell$ is nondecreasing, so unemployed workers search toward ℓ only above some level of human capital.

A worker at a quitting threshold is indifferent between quitting and staying employed, so the surplus is zero there. The first result shows that the slope of the surplus is zero as well. The arguments below use derivatives of the stationary worker values, whose regularity I take as given.[30]

Proposition 3 (Value matching and smooth pasting)**.** *Suppose $\bar{x}_u^\ell > 0$ is a quitting threshold: points immediately to its left belong to $\mathcal{S}_\ell$, and points immediately to its right belong to $\mathcal{C}_\ell$. Then value matching and smooth pasting hold:*

$$V_\ell(\bar{x}_u^\ell) = U(\bar{x}_u^\ell),$$
$$\lim_{x \downarrow \bar{x}_u^\ell} \partial_x V_\ell(x) = \partial_x U(\bar{x}_u^\ell).$$

Proof. Both claims concern the surplus D_ℓ at $\bar{x}_u^\ell$. Value matching says that D_ℓ vanishes there, and smooth pasting says that its slope from the continuation side vanishes too. The first claim is immediate. Continuity makes $\mathcal{S}_\ell$ closed, and $\bar{x}_u^\ell$ is a limit point of $\mathcal{S}_\ell$, so $D_\ell(\bar{x}_u^\ell) = 0$.

For smooth pasting, the mean-value theorem shows that the finite continuation-side limit of $\partial_x D_\ell$ equals the right derivative of D_ℓ at $\bar{x}_u^\ell$. This derivative is nonnegative, because $D_\ell \geq 0$ and $D_\ell(\bar{x}_u^\ell) = 0$. Suppose, to reach a contradiction, that it is positive. Then a worker at $\bar{x}_u^\ell$ who delays quitting for a short time h gains from favorable shocks, which carry the worker into the region where the surplus is positive. This gain is of order $\sqrt{h}$, while the flow and discounting costs of the delay are of order h. For small h, delay would therefore beat quitting. The rest of the proof makes this comparison precise.

A positive right derivative gives $c > 0$ and a radius r, with $0 < r < \bar{x}_u^\ell/2$ and $(\bar{x}_u^\ell, \bar{x}_u^\ell + r) \subseteq \mathcal{C}_\ell$, such that $D_\ell(x) \geq c(x - \bar{x}_u^\ell)$ for $x \in (\bar{x}_u^\ell, \bar{x}_u^\ell + r)$. The delay strategy is as follows. Let τ_r be the date human capital leaves $(\bar{x}_u^\ell - r, \bar{x}_u^\ell + r)$ and τ_{δ_ℓ} the date of exogenous separation from ℓ. Starting at $\bar{x}_u^\ell$, the worker postpones quitting and does not search until the earliest of h, τ_r, τ_{δ_ℓ}, and τ_δ. After

[29] Sufficient conditions on the primitives are available from the author upon request.

[30] I take as given $U \in C^3((0,\infty))$ and $V_\ell \in C^3(\mathcal{C}_\ell \cap (0,\infty))$, with finite continuation-side derivative limits at finite boundaries and a zero derivative where human capital is reflected at zero.

the delay, the worker continues optimally, so continuation is valued at the corresponding worker value, and labor-force exit pays zero. Let J_h be the value of this strategy. Since the strategy is feasible, $J_h \le V_\ell(\bar{x}_u^\ell)$.

To compare J_h with $U(\bar{x}_u^\ell)$, I split the employment continuation value as $U + D_\ell$. The expected change in U is only $O(h)$, while the surplus carries the gain. I apply Itô's lemma to U up to the end of the delay. On the neighborhood, wages, U, and its derivatives are bounded, and the stopped martingale has zero expectation. Flow income, changes in U, discounting, and exogenous events therefore contribute $O(h)$, and for some finite C,

$$\begin{aligned} J_h - U(\bar{x}_u^\ell) \ge & -Ch \\ & + \mathbb{E}\big[e^{-\rho h} D_\ell(x(h)) \mathbb{1}\{\tau_r > h,\ \min\{\tau_{\delta_\ell}, \tau_\delta\} > h\}\big]. \end{aligned}$$

Dropping the terminal surplus on paths with $\tau_r \le h$ preserves the bound, since that surplus is nonnegative. On the event $\{\tau_r > h,\ \min\{\tau_{\delta_\ell}, \tau_\delta\} > h\}$, the worker is still in state ℓ, and $r < \bar{x}_u^\ell/2$ keeps human capital away from zero. Hence $x(h) - \bar{x}_u^\ell = \mu_\ell h + \sigma_\ell W_h$.

It remains to bound the surplus term from below. The drift contributes only $O(h)$, whereas $\mathbb{E}[W_h^+] = \sqrt{h/(2\pi)}$. Exogenous events have probability $O(h)$, and leaving the neighborhood before h has exponentially small probability, so excluding these paths changes the Brownian gain by $o(\sqrt{h})$. The local surplus bound therefore gives

$$\mathbb{E}\big[e^{-\rho h} D_\ell(x(h)) \mathbb{1}\{0 < x(h) - \bar{x}_u^\ell < r,\ \tau_r > h,\ \min\{\tau_{\delta_\ell}, \tau_\delta\} > h\}\big] \ge \frac{c\sigma_\ell}{\sqrt{2\pi}}\sqrt{h} + o(\sqrt{h}).$$

For small h, the gain of order $\sqrt{h}$ dominates the loss of order h, so $V_\ell(\bar{x}_u^\ell) \ge J_h > U(\bar{x}_u^\ell)$. This contradicts value matching. Hence the continuation-side slope of D_ℓ is zero, which is smooth pasting. □

The next result shows that quitting, when it occurs, is confined to a bounded interval of low human capital. Its proof uses the generator of human capital in state ℓ, $\mathcal{L}_\ell = \mu_\ell \partial_x + (\sigma_\ell^2/2)\partial_{xx}$.

Proposition 4 (Classification of quitting sets)**.** *Suppose increasing differences hold in both sectors. Fix $\ell \in \{i, f\}$ and suppose $\sigma_\ell^2/\sigma_u^2 > 1 - a_\ell/\bar{a}$. There is a tolerance $\epsilon_\ell > 0$, which does not depend on $\kappa_\ell^{-\ell}$, such that, if $\kappa_u^{-\ell}/\kappa_\ell^{-\ell} > \sigma_\ell^2/\sigma_u^2 - \epsilon_\ell$, the quitting set is either empty or a bounded lower interval $[0, \bar{x}_u^\ell]$. Thus the continuation set is connected, and when quitting occurs, its threshold is unique.*

Observe that the volatility restriction holds automatically in a sector whose wage slope equals $\bar{a}$, so it has content only in the sector with the lower slope. The search-cost restriction is best read as a bound on search from employment: searching toward the other sector cannot be too costly for an employed worker relative to an unemployed one, once costs are scaled by the variance ratio. Finally, note that the proposition separates two cases at the bottom of the state space. A zero threshold means that workers quit at zero human capital, whereas an empty quitting set means that human capital is reflected there.

Proof. Fix $\ell \in \{i, f\}$. There are two claims: $\mathcal{S}_\ell$ is a lower interval, and it is bounded. The first follows from increasing differences. Because $D_\ell \geq 0$ is nondecreasing, $D_\ell(y) = 0$ implies $D_\ell(x) = 0$ for every $x \leq y$. For the second, the proof constructs a comparison function Γ_ℓ that is nonpositive at every positive quitting point and positive above an explicit level X_ℓ. No quitting point can then exceed X_ℓ.

The comparison function combines the two HJB equations. Write $F_\ell = \mathbb{1}\{\ell = f\}F$ for the employment flow cost. Substituting $V_\ell = U + D_\ell$ into the employment HJB and eliminating U'' with the unemployment HJB gives

$$\Gamma_\ell(x) = a_\ell x - F_\ell - \frac{\sigma_\ell^2}{\sigma_u^2} b + \left(\frac{\sigma_\ell^2}{\sigma_u^2} - 1\right)(\rho + \delta)U(x) + \left(\mu_\ell - \frac{\sigma_\ell^2}{\sigma_u^2}\mu_u\right)U'(x) + \left(\frac{\kappa_u^{-\ell}}{\kappa_\ell^{-\ell}} - \frac{\sigma_\ell^2}{\sigma_u^2}\right)\frac{\left([D_{-\ell}(x) - \Psi_{-\ell}]^+\right)^2}{2\kappa_u^{-\ell}}. \tag{A.5}$$

The last term contains $[D_{-\ell} - \Psi_{-\ell}]^+$, the net gain in the unemployed worker's search rule toward the other employment state, equation (5). By Lemma 2 and increasing differences, this gain is nondecreasing and Lipschitz with constant $\bar{a}/(\rho + \delta)$.

At a positive quitting point y, the smooth function U touches V_ℓ from below, and a short feasible continuation cannot improve on $U(y)$. I therefore use U as the test function in the dynamic-programming supersolution inequality. Optimizing search and substituting the unemployment HJB then gives $\Gamma_\ell(y) \leq 0$.

Two bounds on the unemployment value control Γ_ℓ. Lemma 1 gives $b/(\rho+\delta) \leq U(x) \leq \bar{a}x/(\rho+\delta) + B$, where B does not depend on search costs. Since $U'' \geq 0$, $\mu_u \leq 0$, and $0 \leq U' \leq \bar{a}/(\rho + \delta)$, the unemployment HJB also bounds the value of search toward the other employment state:

$$0 \leq \frac{\left([D_{-\ell}(x) - \Psi_{-\ell}]^+\right)^2}{2\kappa_u^{-\ell}} \leq (\rho + \delta)U(x) - b - \mu_u U'(x) \leq \bar{a}x + K, \tag{A.6}$$

with $K = (\rho + \delta)B - b - \mu_u\bar{a}/(\rho + \delta)$. The construction of B in Lemma 1 gives $(\rho + \delta)B > b$, so $K > 0$. The estimate extends to zero by continuity.

Bounded quitting sets. The bound on quitting comes from the linear growth of Γ_ℓ. The volatility restriction makes the slope margin $\gamma_\ell = a_\ell - (1 - \sigma_\ell^2/\sigma_u^2)^+\bar{a}$ positive. Define

$$X_\ell = \frac{2\{b + F_\ell + [1 + (1 - \sigma_\ell^2/\sigma_u^2)^+]K\}}{\gamma_\ell}, \qquad \epsilon_\ell = \frac{\gamma_\ell}{2\bar{a}}.$$

Neither constant depends on $\kappa_\ell^{-\ell}$, so the search-cost restriction is not circular. That restriction makes the coefficient of the last term in Γ_ℓ larger than $-\epsilon_\ell$. The sign of the coefficient on U determines which bound I use. I use $U \geq b/(\rho + \delta)$ when $\sigma_\ell \geq \sigma_u$ and $U \leq \bar{a}x/(\rho + \delta) + B$ when

$\sigma_\ell < \sigma_u$. For the drift term, $\mu_\ell \geq \mu_u$ and $0 \leq U' \leq \bar{a}/(\rho+\delta)$ give

$$\left(\mu_\ell - \frac{\sigma_\ell^2}{\sigma_u^2}\mu_u\right) U'(x) \geq \left(1 - \frac{\sigma_\ell^2}{\sigma_u^2}\right)^+ \mu_u \frac{\bar{a}}{\rho+\delta}.$$

The negative drift contribution when $\sigma_\ell < \sigma_u$ is included in K. Together with equation (A.6), these estimates give

$$\Gamma_\ell(x) \geq (\gamma_\ell - \bar{a}\epsilon_\ell)x - b - F_\ell - \left[(1 - \sigma_\ell^2/\sigma_u^2)^+ + \epsilon_\ell\right]K.$$

Since $\epsilon_\ell \leq \gamma_\ell/(2\bar{a}) \leq 1/2$, the right side is positive for $x > X_\ell$. Every positive quitting point satisfies $\Gamma_\ell \leq 0$ and therefore lies in $[0, X_\ell]$. If $\mathcal{S}_\ell$ is nonempty, it is closed and downward closed, so it has a largest point $\bar{x}_u^\ell \in [0, X_\ell]$ and $\mathcal{S}_\ell = [0, \bar{x}_u^\ell]$. Otherwise $\mathcal{S}_\ell$ is empty. □

Upper-tail behavior. At high human capital, wages dominate benefits and fixed costs, so a strict formal wage advantage governs behavior there. Values then grow linearly, while surpluses and active search rates grow only like $\sqrt{x}$. With quadratic search costs, the optimized search term in the HJB is proportional to the square of the positive net value gain. A gain of order $\sqrt{x}$ can therefore offset a wage gap of order x.

Proposition 5 (Upper-tail values and search policies)**.** *Suppose $a_f > a_i$ and $\kappa_i^f(a_f - a_i) < a_f\kappa_u^f$, and let $d = \sqrt{2\kappa_i^f(a_f - a_i)}$. Then there is a unique $q^\star > 0$ satisfying*

$$\frac{(q^\star)^2}{2\kappa_u^i} + \frac{(q^\star + d)^2}{2\kappa_u^f} = a_f, \tag{A.7}$$

and the following limits hold as $x \to \infty$.

(a) Values grow at the formal slope, and surpluses grow like $\sqrt{x}$:

$$\frac{U(x)}{x},\ \frac{V_i(x)}{x},\ \frac{V_f(x)}{x} \longrightarrow \frac{a_f}{\rho+\delta}, \tag{A.8}$$

$$\frac{V_i(x) - U(x)}{\sqrt{x}} \longrightarrow q^\star, \qquad \frac{V_f(x) - U(x)}{\sqrt{x}} \longrightarrow q^\star + d. \tag{A.9}$$

Hence $(V_f - V_i)/\sqrt{x} \to d$, and both employment continuation sets contain an upper tail.

(b) Search from formal toward informal employment stops: $\eta_f^i(x) = 0$ for all sufficiently large x. Every other search intensity grows like its net gain divided by its cost: $\eta_u^i(x)/\sqrt{x} \to q^\star/\kappa_u^i$, $\eta_u^f(x)/\sqrt{x} \to (q^\star + d)/\kappa_u^f$, and $\eta_i^f(x)/\sqrt{x} \to d/\kappa_i^f$.

Note that the constant d has a direct interpretation. In the upper tail, an informal worker earns the lower wage but searches for formal jobs. To first order, the optimized search term in the HJB, $d^2x/(2\kappa_i^f)$, offsets the wage gap $(a_f - a_i)x$. This is why informal employment shares the

formal slope $a_f/(\rho+\delta)$. The search-cost restriction, in turn, keeps the limit of the normalized informal surplus $(V_i - U)/\sqrt{x}$ strictly positive. Without it, the informal surplus could grow more slowly than $\sqrt{x}$, and unemployed search toward informal jobs would become negligible relative to search toward formal ones.

Proof. The proof identifies three numbers: the common limit $\mathcal{A}$ of the ratios $W_\ell(x)/x$, and the limits p_i and p_f of the normalized surpluses $D_i(x)/\sqrt{x}$ and $D_f(x)/\sqrt{x}$. It proceeds in three steps. First, I bound surpluses by a constant times $1+\sqrt{x}$, so the normalized surpluses are bounded. Convexity and the value bounds then give convergence of the value ratios, and the surplus bound makes their limits coincide at $\mathcal{A}$. Second, passing to the limit in the HJB system along sequences $x_n \to \infty$ yields an algebraic system for $\mathcal{A}$, p_i, and p_f. Third, solving that system under the wage ordering and the search-cost restriction gives $(\rho+\delta)\mathcal{A} = a_f$, $p_i = q^\star$, and $p_f = q^\star + d$. Neither assumption is used in the first two steps.

Values and surpluses. The first step bounds the surpluses. For $x > 0$, convexity gives a supporting slope $s_x \in [\partial_x^- U(x), \partial_x^+ U(x)]$. The line $U(x) + s_x(z-x)$ touches U from below and has zero curvature, so the unemployment supersolution inequality gives

$$\frac{D_i(x)^2}{2\kappa_u^i} + \frac{([D_f(x) - \Psi_f]^+)^2}{2\kappa_u^f} \le (\rho+\delta)U(x) - b - \mu_u s_x \le \bar{a}x + C_0$$

for some finite C_0. The last inequality uses $0 \le s_x \le \bar{a}/(\rho+\delta)$, the upper bound on U in Lemma 1, and $\mu_u \le 0$. Since $D_i, D_f \ge 0$ and $\Psi_f \ge 0$, it follows that, for some finite C,

$$D_i(x) + D_f(x) \le C(1+\sqrt{x}), \qquad x \ge 0. \tag{A.10}$$

Continuity extends this bound to zero.

With this bound, the value ratios converge. For each W_ℓ, convexity makes $[W_\ell(x) - W_\ell(0)]/x$ nondecreasing, and Lemma 2 keeps it below $\bar{a}/(\rho+\delta)$. Thus $W_\ell(x)/x$ has a finite limit. Since equation (A.10) gives $D_\ell(x)/x \to 0$, the three limits coincide. Their common value $\mathcal{A}$ satisfies $\mathcal{A} \le \bar{a}/(\rho+\delta)$.

The second step passes to the limit in the HJB system. Equation (A.10) also makes $D_\ell(x)/\sqrt{x}$ bounded, so along any sequence $x_n \to \infty$ a subsequence, not relabeled, satisfies $D_\ell(x_n)/\sqrt{x_n} \to p_\ell \ge 0$ for $\ell \in \{i, f\}$. To extend these limits to neighborhoods of x_n, I use the Lipschitz bound in Lemma 2. On every bounded interval of shifts z, the ratios $W_\ell(x_n+z)/x_n$ and $D_\ell(x_n+z)/\sqrt{x_n}$ differ from their values at $z=0$ by terms of order x_n^{-1} and $x_n^{-1/2}$, respectively. The convergence to $\mathcal{A}$ and p_ℓ is therefore locally uniform in z.

On each fixed compact interval, the surpluses are bounded, so the maximizing search intensities in equation (5) are bounded as well. The short-horizon dynamic-programming comparison then applies, giving the unemployment equation and the employment supersolution inequalities, with equality on continuation.

I translate these inequalities by x_n and divide by x_n. The normalized search gains, and hence their squares, converge locally uniformly. Benefits and flow costs contribute b/x_n and F/x_n, separation contributes $\delta_\ell D_\ell/x_n$, and the entry cost enters the normalized gain as $\Psi_f/\sqrt{x_n}$. All of these terms vanish.

Stability of viscosity inequalities, applied to the constant limiting value profile, then gives[31]

$$\begin{aligned}(\rho+\delta)\mathcal{A} &= \frac{p_i^2}{2\kappa_u^i} + \frac{p_f^2}{2\kappa_u^f},\\ (\rho+\delta)\mathcal{A} &\geq a_i + \frac{([p_f - p_i]^+)^2}{2\kappa_i^f},\\ (\rho+\delta)\mathcal{A} &\geq a_f + \frac{([p_i - p_f]^+)^2}{2\kappa_f^i}.\end{aligned} \tag{A.11}$$

If $p_\ell > 0$, the employment obstacle is inactive on every translated compact set for sufficiently large n, so the corresponding inequality holds with equality.

The third step solves this system, and only here do the assumptions enter. Under the wage ordering, $\bar{a} = a_f$. The bound on $\mathcal{A}$ and the formal-employment inequality in (A.11) therefore give $(\rho+\delta)\mathcal{A} = a_f$ and $p_i \leq p_f$. Since $a_f > 0$, the unemployment equation then implies $p_f > 0$. The search-cost restriction rules out $p_i = 0$. In that case, the unemployment equation would give $p_f^2 = 2\kappa_u^f a_f$, and the informal-employment inequality would require $\kappa_i^f(a_f - a_i) \geq a_f\kappa_u^f$, the opposite of the search-cost restriction. Thus $p_i, p_f > 0$, and both employment inequalities hold with equality.

The informal-employment equation now gives $p_f - p_i = d$. With $q^\star = p_i$, the unemployment equation becomes (A.7). Its left-hand side is continuous and strictly increasing in $q^\star \geq 0$, equals $d^2/(2\kappa_u^f)$ at zero, and tends to infinity. Hence a unique positive solution exists exactly when $d^2 < 2a_f\kappa_u^f$, which is the search-cost restriction. Every convergent subsequence of the bounded vector of normalized surpluses has the limit $(q^\star, q^\star + d)$, so the full vector converges. Since both limits are positive, both surpluses are eventually positive, and both continuation sets contain an upper tail.

Search policies. Since $V_f - V_i \sim d\sqrt{x}$ with $d > 0$, the net gain from moving from formal to informal employment is eventually negative, so $\eta_f^i(x) = 0$ for all sufficiently large x. Dividing the remaining search rules in equation (5) by $\sqrt{x}$, and using $\Psi_f/\sqrt{x} \to 0$, gives the three stated limits. □

A.2 Worker Distributions and Population Accounting

This subsection turns the worker policies of Appendix A.1 into worker distributions. Throughout, the policies are the stationary ones characterized there, and I maintain increasing differences.

[31] See Lemma 6.1 and Remarks 6.2–6.4 of Crandall et al. (1992).

The first result shows that each worker's state is well defined under these policies, the second that the population is conserved, and the third constructs the unique stationary distribution used in the equilibrium and sorting arguments.

The worker process. Each worker's state at date t is the pair $(\ell(t), x(t))$ of labor-market state and human capital, evolving as in Section 2. Within a spell in state ℓ, human capital follows the reflected diffusion $\mathrm{d}x = \mu_\ell \,\mathrm{d}t + \sigma_\ell \,\mathrm{d}W + \mathrm{d}L$. The worker moves to ℓ' at rate $\eta_\ell^{\ell'}(x)$, separates from employment at rate δ_ℓ (with $\delta_u = 0$), quits on first contact with $\mathcal{S}_\ell$, and leaves the labor force at rate δ. Each departing worker is replaced by an unemployed entrant whose human capital is drawn from g. Write $m_\ell(\mathrm{d}x, t)$ for the mass of workers in state ℓ, the measure counterpart of the density $m_\ell(x, t)$.

Three facts from Appendix A.1 govern these movements. First, search intensities grow at most linearly in human capital. Each value gain in the search rules (5) is a difference of two values whose slopes lie in $[0, \bar{a}/(\rho + \delta)]$ by Lemma 2, so every intensity satisfies $\eta_\ell^{\ell'}(x) \leq C(1 + x)$ for a finite C. Second, by increasing differences, each quitting set is a closed lower interval. Search toward a quitting set is zero, because the destination value equals U there while the origin value is at least U and entry costs are nonnegative. A quitting set equal to the whole state space therefore leaves its sector empty. Otherwise $\mathcal{S}_\ell$ is empty or $[0, \bar{x}_u^\ell]$. Third, workers enter employment only in the continuation set and human capital moves continuously, so an employed worker quits exactly when human capital first falls to $\bar{x}_u^\ell$.

Two conventions complete the description. Initial employment in a quitting set is reassigned to unemployment at date zero. Quitting at zero makes zero an absorbing employment boundary, whereas a sector without quitting keeps reflection there.

Lemma 3 (Worker histories and transition counts)**.** *Under these policies, each worker's state is well defined at every date, and transitions do not accumulate in finite time. If the entrant density and the initial distribution have finite first moments, then on every bounded time interval the population's expected human capital is finite, and so are the expected numbers of search transitions, separations, and quits.*

Proof. The proof constructs the worker state until human capital first reaches a level R, shows that this date tends to infinity with R, and then counts events.

Fix an initial state, suppress labor-force exit, and let τ_R be the first date at which $x(t) \geq R$. Before τ_R, the linear bound caps the total search rate, $\sum_{\ell'} \eta_{\ell(t)}^{\ell'}(x(t)) \leq C(1 + R)$, and separations occur at rate at most $\max_\ell \delta_\ell$. Search and separation arrivals before τ_R are therefore dominated by a Poisson process with a finite rate, so only finitely many occur on a bounded interval. Quits cannot outnumber employment entries plus any initial quit. The worker state is thus uniquely defined on $[0, \tau_R)$.

To extend the construction to all dates, I show that $\tau_R \to \infty$ as $R \to \infty$. Applied to the path stopped at τ_R, the reflected-path representation in the proof of Lemma 1 gives $\sup_{s \leq t} x(s) \leq$

$x + 2\bar{\mu}t + 2\sup_{s\le t}|M(s)|$. Doob's and Markov's inequalities then yield

$$\mathbb{P}(\tau_R \le t) \le \frac{x + 2\bar{\mu}t + 4\bar{\sigma}\sqrt{t}}{R} \longrightarrow 0 \qquad \text{as } R \to \infty.$$

Hence $\tau_R \to \infty$, so the worker state is well defined at every date, with finitely many transitions on bounded intervals. Labor-force exit and replacement restart it from an entrant draw.

Fix T and assume the moment conditions. I next show that the numbers of search transitions, separations, and quits on $[0, T]$ have finite expectations. Let $N(T)$ be the number of search transitions on $[0, T]$. Its expectation is the search rate integrated over the population and over time, so the linear bound gives

$$\mathbb{E}[N(T)] = \int_0^T \sum_\ell \int \sum_{\ell'} \eta_\ell^{\ell'}(x)\, m_\ell(\mathrm{d}x, s)\, \mathrm{d}s \le C \int_0^T \sum_\ell \int (1 + x)\, m_\ell(\mathrm{d}x, s)\, \mathrm{d}s.$$

At date s, the population consists of surviving initial workers and surviving entrants from $[0, s]$. Its total mass is therefore at most $\mathbf{M}(0) + \delta s$. By equation (A.3), each worker's expected human capital rises by at most $2\bar{\mu}s + 2\bar{\sigma}\sqrt{s}$ over an interval of length at most s. Therefore

$$\sum_\ell \int x\, m_\ell(\mathrm{d}x, s) \le \sum_\ell \int x\, m_\ell(\mathrm{d}x, 0) + \delta s \int_0^\infty x g(x)\, \mathrm{d}x + \big[\mathbf{M}(0) + \delta s\big]\big(2\bar{\mu}s + 2\bar{\sigma}\sqrt{s}\big).$$

The right side is bounded for $s \in [0, T]$ because the initial distribution and g have finite first moments. Together with the population-mass bound, this gives $\mathbb{E}[N(T)] < \infty$. The expected number of separations is at most $\max_\ell \delta_\ell \int_0^T \mathbf{M}(s)\, \mathrm{d}s$, which is finite. Quits cannot outnumber employment entries plus those initially employed, so their expected number is finite as well. □

Population accounting. The next result shows that these transitions conserve the population. They move workers across states without creating or destroying them, so only labor-force exit and entry change the total.

Proposition 6 (Population conservation)**.** *Suppose the entrant density and the initial distribution have finite first moments, and let* $\mathbf{M}(t)$ *be the total population, the sum of the state masses* $\mathbf{M}_\ell(t)$*. Then the worker distributions satisfy the forward equations* (7) *in the weak sense, and*

$$\mathbf{M}(t) = 1 + [\mathbf{M}(0) - 1]e^{-\delta t}.$$

In particular, a unit initial population remains one.

Proof. Both claims follow from accounting for each state's inflows and outflows. Fix a state ℓ and a smooth compactly supported test function φ, with $\varphi' = 0$ at a reflecting boundary. I apply Itô's formula, spell by spell, to $\varphi(x(t))$ while the worker is in ℓ, and take expectations. The finite expected transition counts in Lemma 3 make the resulting inflow and outflow terms well defined.

Movement of human capital within ℓ contributes the generator, and each transition into or out of ℓ contributes φ evaluated at the human capital where it occurs:

$$\int \varphi \, m_\ell(\mathrm{d}x, t) - \int \varphi \, m_\ell(\mathrm{d}x, 0) = \int_0^t \int \mathcal{L}_\ell \varphi \, m_\ell(\mathrm{d}x, s) \, \mathrm{d}s + \mathrm{In}_\ell(\varphi, t) - \mathrm{Out}_\ell(\varphi, t). \tag{A.12}$$

The outflow term integrates φ against search at rates $\eta_\ell^{\ell'}$, separation at rate δ_ℓ, labor-force exit at rate δ, and quits. The inflow term integrates φ against search from other states and, for unemployment, against separations, quits, and entrants arriving at rate δg. These are the flows in equation (7), with quits counted by their expected measures $q_\ell(\mathrm{d}x, \mathrm{d}s)$, so equation (A.12) is the weak form of equation (7).

Localization and the finite flows of Lemma 3 allow me to use the constant test function $\varphi = 1$. I then add over states. The generator term vanishes because $\mathcal{L}_\ell 1 = 0$. Every search, separation, and quit is an outflow from one state and an inflow into another, so these terms cancel in the sum, and only entry and exit remain:

$$\mathbf{M}(t) - \mathbf{M}(0) = \delta \int_0^t \big[1 - \mathbf{M}(s)\big] \, \mathrm{d}s.$$

Solving this equation gives the stated law. □

Stationary worker distribution. Under stationary policies, the population renews itself: each departing worker is replaced by an entrant drawn from g. Because workers leave at the constant rate δ, their time since entry has an exponential distribution in the stationary population. The stationary law averages the state distributions reached by entrants over this age distribution and does not depend on the initial population.

Proposition 7 (Stationary worker distribution)**.** *Under the stationary policies, the worker distribution has the following properties.*

(a) There is a unique stationary distribution, with sectoral densities and unit total mass.

(b) If the entrant density has a finite first moment, so does the stationary distribution. It satisfies the stationary forward equations in the weak sense. Aggregate search, separation, and quitting flows are finite, and total employment output is strictly positive and finite.

(c) Under the moment condition in (b), the densities are continuous, and twice continuously differentiable away from quitting thresholds. As human capital approaches a threshold from above, each density's derivative and flux have finite limits. The fluxes vanish at infinity. Let Q_ℓ be the aggregate stationary quit rate from sector ℓ. At a quitting threshold, the employment density vanishes and the quit rate equals the flux into the threshold:

$$\lim_{x \downarrow \bar{x}_u^\ell} m_\ell(x) = 0, \qquad Q_\ell = -\lim_{x \downarrow \bar{x}_u^\ell} \mathcal{J}_\ell[m_\ell](x) = \frac{\sigma_\ell^2}{2} \lim_{x \downarrow \bar{x}_u^\ell} m_\ell'(x) \geq 0.$$

In a sector without quitting, the flux vanishes at zero, $\mathcal{J}_\ell[m_\ell](0) = 0$.

Proof. The stationary law is the distribution of an entrant's state at an independent exponential age. The proof first shows that this age average is invariant and unique, which gives (a). Under the moment condition, I average the human-capital bound over entrants and ages to establish a finite first moment. Lemma 3 then gives finite flows, allowing me to apply Proposition 6 to the invariant law. A separate argument gives positive output, completing (b). Finally, the weak stationary equations yield the regularity and boundary conditions in (c).

Construction and uniqueness. Let $P_t(B)$ be the probability that an entrant belongs to a measurable set B of worker states at age t in the process without labor-force exit. I average these probabilities over the exponential age distribution:

$$m(B) = \delta \int_0^\infty e^{-\delta t} P_t(B)\, \mathrm{d}t. \tag{A.13}$$

This law has unit mass. To show invariance, I take m as the initial distribution and restore labor-force exit and replacement. At date $h > 0$, workers who have not been replaced during $[0, h]$ contribute

$$e^{-\delta h}\delta \int_0^\infty e^{-\delta t} P_{t+h}(B)\, \mathrm{d}t = \delta \int_h^\infty e^{-\delta t} P_t(B)\, \mathrm{d}t.$$

Workers whose most recent replacement occurred during this interval contribute $\delta \int_0^h e^{-\delta t} P_t(B)\, \mathrm{d}t$. Their sum is $m(B)$, proving invariance.

To prove uniqueness, I couple two workers from arbitrary initial distributions using a common replacement clock. At their first replacement, I use the same entrant draw and subsequent shocks for both workers. Their states coincide thereafter, so they can differ at date h only if no replacement has occurred, an event with probability $e^{-\delta h}$. Their probabilities of belonging to any set B therefore differ by at most $e^{-\delta h}$. If both initial distributions are invariant, these probabilities do not change with h. Letting $h \to \infty$ therefore shows that the two distributions coincide.

To establish densities, I use the occupation-time formula in the process without labor-force exit. The quadratic variation of human capital accumulates at rate at least $\min_j \sigma_j^2 > 0$, so a worker spends zero time in any Lebesgue-null set of human-capital levels. The age mixture integrates over time and entrant histories, so it assigns every such set zero stationary mass in each labor-market state. Hence the stationary law has densities m_ℓ, completing (a).

Moments, flows, and output. Under the moment condition in (b), average equation (A.3) over entrants and ages:

$$\sum_\ell \int_0^\infty x m_\ell(x)\, \mathrm{d}x \le \int_0^\infty x g(x)\, \mathrm{d}x + C\delta \int_0^\infty e^{-\delta t}(t + \sqrt{t})\, \mathrm{d}t < \infty.$$

The invariant law thus has a finite first moment, and Lemma 3 gives finite search, separation, and

quit flows. Stationarity makes expected quit counts proportional to interval length, which defines finite stationary quit-rate measures. These finite flows allow me to apply Proposition 6 to the invariant law. Since this law is stationary, the left side of equation (A.12) vanishes, giving the stationary weak forward equations.

Employment output is finite by the first-moment bound. To prove that it is positive, I first show that an employment-entry intensity is positive on an open interval. If both search intensities out of unemployment vanished everywhere, then $U = b/(\rho + \delta)$. The formal-value floor in Lemma 1(b) would then make $V_f - U - \Psi_f > 0$ at large x, contradicting zero formal search. Hence an employment-entry intensity is positive on an open interval in $(0, \infty)$, where the destination's surplus is positive and entry does not trigger quitting.

Choose a compact interval inside this region on which the entry rate is bounded away from zero. I restrict attention to entrants whose human capital lies in a bounded set with positive probability under g. Positive unemployment volatility gives these entrants a positive probability of reaching the interval along a bounded path before any search arrival. Locally bounded rates then give a positive probability of entering employment and remaining briefly in the destination's continuation region, with human capital bounded away from zero. Averaging this event over ages gives strictly positive employment output, completing (b).

Density regularity and boundary conditions. Each employment sector either has a reflecting boundary at zero without quitting, has an absorbing threshold $\bar{x}_u^\ell \geq 0$, or retains no workers and has zero density. In the absorbing case, all quits occur at the threshold, at rate Q_ℓ. I extend each employment density by zero below its quitting threshold.

The stationary weak equations express each flux's distributional derivative as a sum of integrable flow terms and finite quit measures. The fluxes therefore have locally bounded variation on $(0, \infty)$. By equation (6), $\frac{\sigma_\ell^2}{2} m_\ell' = \mu_\ell m_\ell - \mathcal{J}_\ell[m_\ell]$. The right side is locally integrable, so the densities are locally absolutely continuous. Away from quitting thresholds, continuity of the search policies and entrant density g gives continuous flux derivatives. The flux-density relation then gives twice continuously differentiable densities. The same relation shows that the density derivatives have finite one-sided limits wherever the fluxes do. Near zero, integrability of the flows gives finite limits for the fluxes and hence for the density derivatives as $x \downarrow 0$.

With these limits in hand, I obtain the threshold conditions by matching the boundary contribution from the stationary interior equation to the quit outflow in the weak equation. For a smooth test function φ, integration by parts over $(\bar{x}_u^\ell, \infty)$ gives the interior equation's lower-boundary contribution

$$\lim_{x \downarrow \bar{x}_u^\ell} \left[\varphi(x) \mathcal{J}_\ell[m_\ell](x) + \frac{\sigma_\ell^2}{2} \varphi'(x) m_\ell(x) \right].$$

The quit outflow in equation (A.12) gives $-Q_\ell \varphi(\bar{x}_u^\ell)$ instead. The test function's value and derivative at the threshold can be chosen independently. Matching the coefficient of φ' gives the zero density limit because $\sigma_\ell > 0$, while matching the coefficient of φ gives the quit-flux identity. Since

the density is nonnegative and has zero limit at the threshold, its derivative limit from above is nonnegative. The argument applies equally at a threshold of zero. Without quitting, zero reflects and $\mathcal{J}_\ell[m_\ell](0) = 0$.

I next account for the quit inflows into unemployment. Testing its weak equation with $\varphi_u'(0) = 0$ and arbitrary $\varphi_u(0)$ gives $\mathcal{J}_u[m_u](0) = \sum_{\ell:\mathcal{S}_\ell=\{0\}} Q_\ell$. Quits at zero thus generate a boundary inflow, even though each unemployed worker's path reflects there. At a positive quitting threshold, the incoming quit flow makes the unemployment flux jump upward by the quit rate, while its density remains continuous. Integration by parts gives the stationary flux equations.

Finally, I show that each flux vanishes at infinity. Finite transition flows make $\partial_x \mathcal{J}_\ell[m_\ell]$ integrable on an upper tail, so the flux has a finite limit there. Integrability of m_ℓ gives a sequence $R_n \to \infty$ with $m_\ell(R_n) \to 0$. I integrate the flux-density relation from a fixed point to R_n and divide by R_n. After division, the density and integrated-mass terms tend to zero, so the average flux tends to zero as well. Because the flux has a limit at infinity, its averages converge to that same limit, which must therefore be zero. □

Finally, Proposition 6 gives the population accounting for the evolving worker distributions in the main text under the stationary policies characterized above. Integrating the generator terms by parts gives the interior equations in (7). The absorbing employment boundary and the quit term q_ℓ describe the same outflow into unemployment, so this outflow is counted once.

A.3 Stationary Equilibrium

This subsection proves Theorem 1 and then gives conditions under which comparative advantage sorts workers with high human capital into formal employment.

Recall that zero profits alone set wages (2), so workers solve their problem at known wages. The proof follows this sequence: wages determine worker values and policies, these policies determine the stationary distribution, and the distribution determines labor demand and the transfer. It suffices to show that each step has exactly one outcome. As in Appendix A.2, I maintain increasing differences.[32]

Proof of Theorem 1. By condition (i) of Definition 1, wages are those of equation (2), with $a_f = A/(1+\tau)$ and $a_i = A(1-\alpha_0)$. Both are positive because $\alpha_0 < 1$.

Consider first the workers. Each value is the supremum of lifetime payoffs over plans, so U, V_i, and V_f are uniquely determined, and Lemma 1 makes them finite. Let π follow the search rules (5) and quit on first contact with $\mathcal{S}_\ell$. To establish admissibility, I use Lemma 3 to construct the resulting worker history and rule out infinitely many transitions in finite time. The search

[32] The replacement argument for existence and uniqueness of the stationary distribution does not require a single quitting threshold. Given well-defined stationary policies, it applies to any closed quitting set, with quitting at first contact.

intensities are predictable, and the quitting dates are stopping times. The search rules and surplus bound (A.10) bound quadratic search costs by a constant times $1 + x$. Together with the human-capital moment bound (A.3), these estimates make discounted income, search costs, and entry costs integrable. Hence π is admissible.

It remains to show that π attains the worker values. Apply Itô's formula to the discounted worker value, spell by spell, stopping at labor-force exit, a finite horizon, or a human-capital cutoff. The search rates attain the HJB maxima, and the employment equations hold with equality until quitting. Value matching preserves the continuation value at each quit, while the zero derivative at a reflecting boundary removes the regulator term. Taking expectations therefore expresses the initial value as the expected discounted payoff up to the horizon plus the surviving terminal value. The reflected-path bound in the proof of Lemma 3 gives an integrable supremum of human capital on each finite horizon, which permits removal of the spatial cutoff. Transversality in Lemma 1(c) then removes the terminal term as the horizon tends to infinity. Thus $W_\ell(x;\pi) = W_\ell(x)$.

Each search objective is strictly concave, so its maximizing intensity is unique. The values determine the quitting sets, and the maintained first-contact rule determines quitting behavior. Condition (ii) therefore has one solution.

Given these policies, Proposition 7(a) gives a unique stationary distribution with unit mass, which is condition (iii) together with the population constraint. Market clearing then sets each sector's labor demand equal to its employment density. Firms produce under constant returns at zero profits, so they are willing to hire that amount at these wages.

Notice how the finite mean of entrants' human capital enters. Proposition 7(b) gives the stationary distribution a finite first moment. Tax revenue $\tau \int w_f m_f$ is therefore finite, and the budget (8) determines a unique finite transfer. The transfer shifts all values equally and leaves the policies and the distribution unchanged. Every component of the equilibrium is therefore determined, and uniquely so. □

Sectoral comparative advantage. Recall from Section 2.5 that comparative advantage rests on the formal advantage $V_f - V_i$. Pay and stability govern its slope, and the recurring cost F shifts its level. The following proposition gives conditions under which the two values cross exactly once. The wage, human-capital accumulation, and separation comparisons must favor formality. A sufficiently large F makes informality more valuable at the lower boundary of the region where both jobs continue.

Proposition 8 (Sectoral comparative advantage)**.** *Suppose increasing differences hold in both sectors, $a_f \geq a_i$, and*

$$\frac{a_f}{\sigma_f^2} \geq \frac{a_i}{\sigma_i^2}, \qquad \frac{\mu_f}{\sigma_f^2} \geq \frac{\mu_i}{\sigma_i^2}, \qquad \frac{\delta_f}{\sigma_f^2} \leq \frac{\delta_i}{\sigma_i^2},$$

with the separation inequality strict when $a_f = a_i$. If $F > \bar{F}$, where $\bar{F}$ is the explicit bound in equation (A.15) and depends only on the primitives, there is a unique threshold $\bar{x}_i^f$ such that $V_f(x) > V_i(x)$ exactly when $x > \bar{x}_i^f$.

Proof of Proposition 8. Let $E = V_f - V_i$ be the formal advantage. The proof has three steps. First, the cost bound makes E negative at the lowest human capital at which workers keep both jobs. Second, the proposition's three comparisons make the forcing term nondecreasing and unbounded. Third, a minimum principle shows that E can fall only at first and then rises without bound, so it crosses zero once. I begin by collecting the bounds that the argument inherits and deriving the equation for E.

Bounds and the equation for E. Since $a_f \geq a_i$, Lemmas 1 and 2 give convex values with

$$\frac{b}{\rho+\delta} \leq W_j(x) \leq \frac{a_f x + b}{\rho+\delta} + B_0, \qquad 0 \leq W_j' \leq \frac{a_f}{\rho+\delta},$$

where $B_0 = a_f[2\bar{\mu}/(\rho+\delta)^2 + \bar{\sigma}\sqrt{\pi}/(\rho+\delta)^{3/2}]$ is the constant from the proof of Lemma 1. In particular, $D_i(x) \leq a_f x/(\rho+\delta) + B_0$ and $|E'| \leq a_f/(\rho+\delta)$. By increasing differences, each nonempty employment continuation region is an upper interval.

Two facts about high human capital are needed later: formal employment continues there, and the values share the slope $a_f/(\rho+\delta)$. Equation (A.10) bounds the normalized surpluses $D_\ell(x)/\sqrt{x}$, and along any convergent subsequence their limits p_i and p_f satisfy the limit system (A.11), with common value slope $\mathcal{A} \leq a_f/(\rho+\delta)$. The formal-employment inequality of that system gives $(\rho+\delta)\mathcal{A} \geq a_f$ and $p_i \leq p_f$. Hence $(\rho+\delta)\mathcal{A} = a_f$, and the unemployment equation gives $p_f^2 \geq 2a_f/(1/\kappa_u^i + 1/\kappa_u^f) > 0$, so D_f is positive for all sufficiently large x.

Where both jobs continue, subtract σ_f^2/σ_i^2 times the informal HJB from the formal HJB. The informal value's curvature cancels, leaving a second-order equation for E:

$$\mathcal{H}(E) - \mu_f E' - \frac{\sigma_f^2}{2} E'' = \Phi(x), \tag{A.14}$$

where

$$\begin{aligned}
\mathcal{H}(z) &= (\rho+\delta+\delta_f)z + \frac{\sigma_f^2}{\sigma_i^2}\frac{([z-\Psi_f]^+)^2}{2\kappa_i^f} - \frac{([-z]^+)^2}{2\kappa_f^i}, \\
\Phi(x) &= \Big(a_f - \frac{\sigma_f^2}{\sigma_i^2}a_i\Big)x + \Big(\frac{\sigma_f^2}{\sigma_i^2} - 1\Big)(\rho+\delta)V_i(x) - F \\
&\quad + \Big(\mu_f - \frac{\sigma_f^2}{\sigma_i^2}\mu_i\Big)V_i'(x) + \Big(\frac{\sigma_f^2}{\sigma_i^2}\delta_i - \delta_f\Big)D_i(x).
\end{aligned}$$

The function $\mathcal{H}$ collects discounting, separation, and search. In the forcing term Φ, the recurring formal cost enters as $-F$. To derive a sufficient lower bound on F, I bound the remaining terms in Φ from above.

Informal advantage at the bottom. I now show that E is negative at the lower boundary of common continuation. I first bound the region where both jobs are quit by R_0. Then, I construct an upper bound for V_f that equals V_i on $[0, R]$ for some $R > R_0$, while allowing a formal advantage above R. To make this comparison valid, I derive a bound on F that accounts for continuation in

formal employment beyond R through the constant Λ_f. The two constants are:

$$R_0 = \frac{b}{a_i} + \sqrt{\frac{(\sigma_u^2 - \sigma_i^2)^+ a_f}{(\rho+\delta)a_i}}, \qquad \Lambda_f = \frac{\mu_f + \sqrt{\mu_f^2 + 2(\rho+\delta+\delta_f)\sigma_f^2}}{2(\rho+\delta+\delta_f)}.$$

I first show that the common quitting region cannot extend beyond R_0. Suppose both jobs are quit throughout $[0, R]$. On this interval, $V_i = V_f = U$, so the optimized search gains vanish. The unemployment HJB and the informal quitting inequality therefore give

$$a_i x - b + (\mu_i - \mu_u)U' + \frac{\sigma_i^2 - \sigma_u^2}{2}U'' \leq 0.$$

The drift term is nonnegative. If $\sigma_i \geq \sigma_u$, the curvature term is also nonnegative, so $R \leq b/a_i$. Otherwise, integrating $(\sigma_u^2 - \sigma_i^2)U''/2 \geq a_i x - b$ from b/a_i to R, when $R > b/a_i$, and using the slope bound gives $a_i(R - b/a_i)^2 \leq (\sigma_u^2 - \sigma_i^2)a_f/(\rho+\delta)$. In either case, $R \leq R_0$.

With R_0 in hand, I claim that $E \leq 0$ on an interval $[0, R]$ that reaches beyond it. Let

$$\gamma = a_f - \frac{\sigma_f^2}{\sigma_i^2}a_i + \Big(\frac{\sigma_f^2}{\sigma_i^2} - 1\Big)^+ a_f + \Big(\frac{\sigma_f^2}{\sigma_i^2}\delta_i - \delta_f\Big)\frac{a_f}{\rho+\delta}.$$

It is positive. If $\sigma_f < \sigma_i$, then $a_f \geq a_i > a_i\sigma_f^2/\sigma_i^2$. If $\sigma_f > \sigma_i$, the third term is positive. If $\sigma_f = \sigma_i$, either $a_f > a_i$, or the wage slopes are equal and the proposition requires $\delta_i > \delta_f$.

At $x = R_0 + \Lambda_f$, the value and slope bounds give an upper bound for $\Phi(x) + F$. I use this upper bound as the cost threshold $\bar{F}$:

$$\begin{aligned} F > \bar{F} := {} & \Big(a_f - \frac{\sigma_f^2}{\sigma_i^2}a_i\Big)(R_0 + \Lambda_f) + \Big(\mu_f - \frac{\sigma_f^2}{\sigma_i^2}\mu_i\Big)\frac{a_f}{\rho+\delta} \\ & + \Big(\frac{\sigma_f^2}{\sigma_i^2}\delta_i - \delta_f\Big)\Big[B_0 + \frac{a_f(R_0+\Lambda_f)}{\rho+\delta}\Big] + \Big(\frac{\sigma_f^2}{\sigma_i^2} - 1\Big)b \\ & + \Big(\frac{\sigma_f^2}{\sigma_i^2} - 1\Big)^+\big[a_f(R_0+\Lambda_f) + (\rho+\delta)B_0\big]. \end{aligned} \tag{A.15}$$

The value bounds above, using the upper bound on V_i if $\sigma_f \geq \sigma_i$ and the lower bound if $\sigma_f < \sigma_i$, give

$$\Phi(x) \leq \gamma(x - R_0 - \Lambda_f) + \bar{F} - F.$$

Choose $R > R_0$ with $\gamma(R - R_0) < F - \bar{F}$, so that $\Phi(x) < \gamma(x - R - \Lambda_f)$.

To show that $E \leq 0$ on $[0, R]$, I bound the formal value from above by the informal value plus an allowance for continuing in formal employment beyond R. With $\lambda_- = -2(\rho+\delta+\delta_f)\Lambda_f/\sigma_f^2 < 0$,

the allowance is

$$\Xi_R(x) = \begin{cases} 0, & x \le R, \\ \dfrac{\gamma}{\rho+\delta+\delta_f}\left[(x-R) + \dfrac{e^{\lambda_-(x-R)}-1}{-\lambda_-}\right], & x > R. \end{cases}$$

It is nonnegative, has value and derivative zero at R, and grows linearly. Direct substitution gives $(\rho+\delta+\delta_f)\Xi_R - \mathcal{L}_f\Xi_R = \gamma(x-R-\Lambda_f)$ above R, while below R, $\Xi_R = 0$ and $\Phi < 0$. The candidate $V_i + \Xi_R$ dominates both unemployment and informal employment, so its search gain toward informal employment is zero. Multiplying the informal HJB inequality by σ_f^2/σ_i^2 and dropping its nonnegative search term gives

$$\begin{aligned}(\rho+\delta+\delta_f)(V_i+\Xi_R) - \mathcal{L}_f(V_i+\Xi_R) - a_f x + F - \delta_f U \\ \ge -\Phi + (\rho+\delta+\delta_f)\Xi_R - \mathcal{L}_f\Xi_R \ge 0,\end{aligned}$$

so $V_i + \Xi_R$ is a supersolution for the formal value. It also dominates what a formal worker receives on quitting or on moving to informal employment. At zero, $\lim_{x\downarrow 0} V_\ell'(x) = 0$, because a positive right derivative would make a short delay in quitting profitable, as in the proof of Proposition 3. A verification argument therefore gives $V_f \le V_i + \Xi_R$, hence $E \le 0$ on $[0, R]$.[33]

It remains to make this ranking strict where both jobs first continue. If $D_i(R) = 0$, then $E(R) \le 0$ also gives $D_f(R) = 0$, and increasing differences would make both jobs quit throughout $[0, R]$, contradicting $R > R_0$. Thus $D_i(R) > 0$, and both employment continuation sets are nonempty. Let x_0 be their lower common boundary. If $x_0 > R$, it is a formal quitting threshold, and $E(x_0) = -D_i(x_0) < 0$. If $x_0 \le R$, then $E(x_0) \le 0$. Equality would require $x_0 < R$ and, by smooth pasting or the zero derivatives at zero, $\lim_{x\downarrow x_0} E'(x) = 0$. Equation (A.14) would then give $\lim_{x\downarrow x_0} E''(x) = -2\Phi(x_0)/\sigma_f^2 > 0$, contradicting $E \le 0$ just above x_0. Hence $E(x_0) < 0$ in every case, and both employment HJBs hold with equality on (x_0, ∞).

The forcing term. This step shows that Φ is nondecreasing and unbounded above. Multiplied by σ_f^2, the proposition's comparisons read $a_f \ge a_i\sigma_f^2/\sigma_i^2$, $\mu_f \ge \mu_i\sigma_f^2/\sigma_i^2$, and $\delta_i\sigma_f^2/\sigma_i^2 \ge \delta_f$. The last two terms of Φ are then nondecreasing, because V_i is convex and D_i is nondecreasing. If $\sigma_f \ge \sigma_i$, the first two terms are nondecreasing as well. If $\sigma_f \le \sigma_i$, rewrite them as

$$\frac{\sigma_f^2}{\sigma_i^2}(a_f - a_i)x + \left(1 - \frac{\sigma_f^2}{\sigma_i^2}\right)(\rho+\delta)\left[\frac{a_f}{\rho+\delta}x - V_i(x)\right],$$

whose bracket is nondecreasing by the slope bound. Hence Φ is nondecreasing in either case.

To see that Φ is unbounded above, suppose first that $a_f > a_i$. The common value slope, the bounded value derivatives, and $D_i(x)/x \to 0$ give $\Phi(x)/x \to (a_f - a_i)\sigma_f^2/\sigma_i^2 > 0$. If instead $a_f = a_i$, the comparison of wage slopes gives $\sigma_f \le \sigma_i$, and the proposition requires $\delta_i\sigma_f^2/\sigma_i^2 > \delta_f$.

[33] Proposition 3 makes V_i continuously differentiable at positive quitting thresholds, and Ξ_R is continuously differentiable at R, so Itô's formula up to the end of a formal spell has no local-time terms. Lemma 1 removes the localizations in time and human capital.

The informal-employment inequality in (A.11), together with $(\rho+\delta)\mathcal{A} = a_f$, gives $p_f \leq p_i$, so every limit of the normalized surpluses has $p_i = p_f > 0$ by the bound $p_f^2 \geq 2a_f/(1/\kappa_u^i + 1/\kappa_u^f)$. Hence $D_i(x) \to \infty$, and the positive coefficient $\delta_i\sigma_f^2/\sigma_i^2 - \delta_f$ makes $\Phi(x) \to \infty$ as well.

A single crossing. Finally, this step shows that E can fall only at first and then rises without bound, so it crosses zero once. The tool is a minimum principle for E'. The search terms are continuously differentiable, with $\mathcal{H}'(E) = \rho+\delta+\delta_f+(\sigma_f^2/\sigma_i^2)\eta_i^f+\eta_f^i > 0$, so differentiating (A.14) gives

$$\mathcal{H}'(E)E' - \mu_f E'' - \frac{\sigma_f^2}{2}E''' = \Phi' \geq 0 \qquad \text{on } (x_0, \infty). \tag{A.16}$$

First, E cannot be bounded above on any upper tail. If $E \leq M$ there, equation (A.14) and the bound on E' give

$$\frac{\sigma_f^2}{2}E'' = \mathcal{H}(E) - \Phi - \mu_f E' \leq \mathcal{H}(M) - \Phi + \frac{\mu_f a_f}{\rho+\delta} \longrightarrow -\infty,$$

which would make E' unbounded below, contradicting its bound.

Now consider where E decreases. A bounded component of $\{E' < 0\}$ whose endpoints lie in common continuation would contain an interior negative minimum of E', where $E'' = 0$ and $E''' \geq 0$. That would make the left side of (A.16) strictly negative. Hence E can decrease only on a component that adjoins x_0 or extends to infinity. An unbounded component would leave E bounded above on a tail, which was just ruled out. So E can decrease only on an initial bounded interval. Beyond it, $E' \geq 0$, and the strong minimum principle applied to (A.16) makes $E' > 0$, because an interior zero would force E' to vanish throughout and make E constant on a tail. If E never decreases, the same argument gives $E' > 0$ throughout common continuation.

The increasing branch cannot remain bounded above, so $E(x) \to \infty$. Since $E(x_0) < 0$ and any initial decline only lowers E further, continuity and the strict increase that follows give a unique $\bar{x}_i^f > x_0$ at which $E = 0$, with $E > 0$ exactly above it in common continuation. Below x_0 there is no crossing. Informal employment cannot be quit there while formal employment continues, because that ordering of the upper continuation regions would give $V_i(x_0) = U(x_0) \leq V_f(x_0)$, contradicting $E(x_0) < 0$. Below common continuation, therefore, either both jobs are quit, with $E = 0$, or only informal employment continues, with $E = -D_i < 0$. The strict formal advantage $V_f > V_i$ thus holds exactly for $x > \bar{x}_i^f$ on the whole state space. Notice that E also vanishes wherever both jobs are quit, so $\bar{x}_i^f$ is the unique crossing of E, not its unique zero. □

The cost bound has a simple reading. Where workers first keep both jobs, formal employment still offers a higher volatility-normalized wage, faster accumulation, and more stability than informal employment, and F must outweigh all three over the range $[0, R_0 + \Lambda_f]$ on which the comparison is made. The first three terms of $\bar{F}$ price these advantages. The remaining terms adjust for unequal employment volatilities and vanish when the volatilities are equal.

Note that the single crossing also shapes search. By the search rule (5), informal-to-formal search is active on a single upper interval, strictly increasing wherever positive, and unbounded as

human capital rises. Its cutoff $\hat{x}_i^f$ lies above $\bar{x}_i^f$ when $\Psi_f > 0$ and coincides with it when $\Psi_f = 0$.

A.4 Sorting and Duration Dependence

This subsection proves Theorem 2 and then derives its consequences for unemployment and informal-to-formal mobility.

The decomposition involves the slopes of search rates and their covariances over survivors, so search rates must be Lipschitz with finite second moments. By Lemma 2 and the search rules (5),

$$|\eta_\ell^{\ell'}(y) - \eta_\ell^{\ell'}(x)| \le \frac{\bar{a}}{(\rho+\delta)\kappa_\ell^{\ell'}}|y-x|, \qquad [\eta_\ell^{\ell'}(x)]^2 \le C(1+x) \tag{A.17}$$

for some finite C. The second bound uses equation (A.10) and $V_\ell \ge U$, which bounds every job-to-job gain by the corresponding gain from unemployment. Since the cohort starts with finite mean human capital, Lemma 3 keeps that mean finite at every finite duration, and with it these second moments.

Hazard decomposition. Recall that the hazard toward ℓ' is the average search intensity of the workers still in the spell. As the spell lengthens, that average can change in two ways. Survivors drift, through accumulation and shocks, toward levels of human capital at which they search more or less, and the mix of survivors shifts as some of them leave. The proof follows both changes through the cohort's forward equation, and an integration by parts sorts them into the theorem's three terms. Unlike the stationary densities of Proposition 7(c), cohort densities change with duration, and for this calculation I take them to be well behaved.[34]

Proof of Theorem 2. Fix an origin ℓ and a duration with $S_\ell(d) > 0$. To cover unemployment with the same calculation, set $\mathcal{C}_u = \mathbb{R}_+$ and $\bar{q}_u = 0$. Let $r_\ell(x)$ denote the rate at which survivors at x leave through search, separation, and labor-force exit.

The cohort receives no inflows, so in the per-state accounting (A.12) only outflows remain: survivors leave at rate r_ℓ inside $\mathcal{C}_\ell$ and through quits at the threshold. For the unnormalized density $S_\ell \bar{m}_\ell$, this gives the forward equation, and integrating it over $\mathcal{C}_\ell$ gives the survival law:

$$\begin{aligned} \partial_d(S_\ell \bar{m}_\ell) &= -S_\ell \partial_x \mathcal{J}_\ell[\bar{m}_\ell] - r_\ell S_\ell \bar{m}_\ell, \\ -\frac{S_\ell'(d)}{S_\ell(d)} &= \mathbb{E}_d[r_\ell] + \bar{q}_\ell(d). \end{aligned}$$

Since the hazard (10) divides search-weighted survivor mass by survival, its derivative combines

[34] Precisely: differentiation under the mass and search-weighted integrals is permitted, the flux terms are integrable with finite limits at the threshold, and $\mathcal{J}_\ell[\bar{m}_\ell]$ and $\eta_\ell^{\ell'}\mathcal{J}_\ell[\bar{m}_\ell]$ vanish at infinity.

the two:

$$\begin{aligned}\partial_d \lambda_\ell^{\ell'}(d) &= \frac{1}{S_\ell(d)} \int_{\mathcal{C}_\ell} \eta_\ell^{\ell'}(x) \partial_d [S_\ell(d) \bar{m}_\ell(x,d)] \, \mathrm{d}x - \frac{S_\ell'(d)}{S_\ell(d)} \lambda_\ell^{\ell'}(d) \\ &= -\int_{\mathcal{C}_\ell} \eta_\ell^{\ell'}(x) \partial_x \mathcal{J}_\ell[\bar{m}_\ell](x,d) \, \mathrm{d}x - \mathrm{Cov}_d(\eta_\ell^{\ell'}, r_\ell) + \bar{q}_\ell(d) \lambda_\ell^{\ell'}(d).\end{aligned}$$

Notice that the last line already has the theorem's shape. The flux term records how survivors move, the covariance records who leaves, and quits enter through $\bar{q}_\ell$. Integrating the flux term by parts moves the derivative onto the search rate:

$$-\int_{\mathcal{C}_\ell} \eta_\ell^{\ell'}(x) \partial_x \mathcal{J}_\ell[\bar{m}_\ell](x,d) \, \mathrm{d}x = \int_{\mathcal{C}_\ell} \partial_x \eta_\ell^{\ell'}(x) \mathcal{J}_\ell[\bar{m}_\ell](x,d) \, \mathrm{d}x - \bar{q}_\ell(d) \eta_\ell^{\ell'}(\bar{x}_u^\ell).$$

The flux vanishes at infinity. At a quitting threshold it equals $-\bar{q}_\ell$, and under reflection at zero it is zero, so the boundary term disappears whenever no one quits. The integral is structural duration dependence.

It remains to reduce the covariance and collect the quit terms. In unemployment, $r_u = \eta_u^i + \eta_u^f + \delta$. The constant δ drops out of the covariance, and with $\bar{q}_u = 0$ this gives equation (12). In employment, the only search is toward the other sector, so $r_\ell = \eta_\ell^{\ell'} + \delta_\ell + \delta$ and the covariance becomes $\mathrm{Var}_d[\eta_\ell^{\ell'}]$. The boundary term removes the search intensity of the workers who quit. The quit term of the survival law raises the weight of those who remain by $\bar{q}_\ell \lambda_\ell^{\ell'}$. Together they form the quit-selection term in equation (13). □

Unemployment duration. Recall that without shocks, human capital cannot rise during unemployment. Survivors who reach zero stay there, so the proof follows individual paths and their survival probabilities.

Proof of Corollary 1. Fix a destination $\ell' \in \{i, f\}$. With $\sigma_u = 0$, a worker starting at human capital x_0 follows $x(d; x_0) = (x_0 + \mu_u d)^+$. The worker remains in unemployment until a search arrival or labor-force exit, so survival conditional on x_0 is

$$S_u(d \mid x_0) = \exp\left[-\int_0^d \left\{ \eta_u^i(x(s; x_0)) + \eta_u^f(x(s; x_0)) + \delta \right\} \mathrm{d}s \right].$$

Write $\mathbb{E}$ for expectation over the initial cohort distribution. Averaging search intensity over survivors gives

$$\lambda_u^{\ell'}(d) = \frac{\mathbb{E}\left[\eta_u^{\ell'}(x(d; x_0)) S_u(d \mid x_0) \right]}{S_u(d)},$$

where $S_u(d) = \mathbb{E}[S_u(d \mid x_0)]$ is cohort survival.

Since $x(d; x_0) \leq x_0$, equation (A.17) bounds the intensity slopes and products along each path by a constant times $1 + x_0$. The finite initial mean therefore permits differentiation under these expectations. The numerator and denominator are locally absolutely continuous, and $S_u(d) > 0$

at every finite duration, so the hazard is locally absolutely continuous as well. Differentiating the ratio and cancelling the constant δ from the covariance gives, for almost every duration,

$$\partial_d \lambda_u^{\ell'}(d) = \mu_u \mathbb{E}_d\big[\partial_x \eta_u^{\ell'}(x) \mathbb{1}\{x > 0\}\big] - \mathrm{Cov}_d\big(\eta_u^{\ell'}(x), \eta_u^i(x) + \eta_u^f(x)\big).$$

The survivor averages include workers at zero, whose drift contribution vanishes. The structural term is nonpositive because $\mu_u \leq 0$ and search is nondecreasing. The covariance is nonnegative by Chebyshev's association inequality, because $\eta_u^{\ell'}$ and $\eta_u^i + \eta_u^f$ are both nondecreasing in human capital. Local absolute continuity then makes the hazard nonincreasing at every duration. □

Informal-to-formal mobility. Recall that informal workers accumulate human capital, so structural duration dependence now pushes the formal hazard up. The proof shows that slow accumulation lets dynamic selection prevail, and it gives an explicit bound on the drift.

Proof of Corollary 2. Fix $D > 0$. There are two claims: the hazard strictly declines on $[0, D]$, and it stays bounded away from zero. Since $V_i - U = \kappa_u^i \eta_u^i$ is nondecreasing, informal continuation is an upper interval. With $\sigma_i = 0$, human capital only rises, along $x(d) = x(0) + \mu_i d$, so no survivor reaches the quitting threshold. Theorem 2 then has no quit term and gives, for almost every duration,

$$\partial_d \lambda_i^f(d) = \mu_i \mathbb{E}_d\big[\partial_x \eta_i^f(x)\big] - \mathrm{Var}_d\big[\eta_i^f(x)\big]. \tag{A.18}$$

Notice that the two forces now pull in opposite directions. By equation (A.17), formal search rises with human capital at slope at most $\bar{a}/[(\rho+\delta)\kappa_i^f]$, so accumulation raises a worker's search at rate at most $\mu_i \bar{a}/[(\rho+\delta)\kappa_i^f]$. It suffices to show that the variance stays above this rate throughout the window. The floor will then follow from the decline.

Selection acts on survivors who still differ in formal search. Workers who search slowly leave slowly, so the natural group to follow is the entrants whose initial search lies below some level η_*. The survival of this group is easy to bound, and under slow accumulation its search stays spread out. Nondegenerate initial human capital, a positive active share, and strictly increasing search on the active branch make initial search $\eta_i^f(x(0))$ nondegenerate. The level η_* can therefore be chosen so that the group has positive mass p and positive dispersion s, where s^2 is the variance of initial search within the group. The drift bound is then chosen so that accumulation neither erases this dispersion nor outweighs it:

$$\bar{\mu}_i(D) = \frac{(\rho+\delta)\kappa_i^f}{\bar{a}} \min\left\{\frac{s}{2D},\ \frac{ps^2}{4} e^{-(\eta_* + s/2)D}\right\}. \tag{A.19}$$

With the group fixed, the bound falls with the horizon and tends to zero, which is the horizon dependence noted in the main text.

Suppose $\mu_i < \bar{\mu}_i(D)$. By the first term of the bound, accumulation raises each worker's search by less than $s/2$ over the window. The group's search therefore keeps a dispersion above $s/2$ and stays below $\eta_* + s/2$. Each member then survives formal-search exits to D with probability at least

$e^{-(\eta_*+s/2)D}$. Separation and labor-force exit strike every worker at the same rate, so they cancel from survivor averages. A variance is the smallest mean squared deviation about any center, and survival probabilities are at most one, so

$$\mathrm{Var}_d\big[\eta_i^f(x)\big] \geq \frac{ps^2}{4} e^{-(\eta_*+s/2)D} > \frac{\mu_i \bar{a}}{(\rho+\delta)\kappa_i^f},$$

where the last inequality is the second term of the bound. Selection therefore outweighs accumulation at almost every duration in $[0, D]$. Since equation (A.17) makes the hazard locally absolutely continuous, the hazard strictly declines.

Finally, the hazard stays positive. Entrants who start out searching keep searching, because their human capital only rises and formal search is nondecreasing, and they survive with positive probability. Hence $\lambda_i^f(D) > 0$, and the decline extends this floor to all of $[0, D]$. □

A.5 Hazard Estimation

This subsection establishes consistency, asymptotic normality, and worker-clustered inference for the interval estimator of Section 3. Appendix C.2 relates its population counterpart to the model's continuous-time hazards.

Estimator and population counterpart. A worker can contribute several spells, and those spells need not be independent, so the arguments below treat workers as the sampling unit. Fix an origin ℓ, a destination $\ell' \neq \ell$, and an interval $I_d = (d - \Delta, d]$ of Δ months, six in the baseline. For worker m, let $Z_{m,Y}$ be the weighted number of spells that move from ℓ to ℓ' during I_d, and $Z_{m,A}$ the weighted number at risk at its start:

$$\begin{aligned} Z_{m,Y} &= \sum_n \omega_{m,n} \mathbf{1}\big\{\ell_{m,n} = \ell,\ T_{m,n} \in I_d,\ \ell'_{m,n} = \ell'\big\}, \\ Z_{m,A} &= \sum_n \omega_{m,n} \mathbf{1}\big\{\ell_{m,n} = \ell,\ T_{m,n} > d - \Delta\big\}. \end{aligned} \tag{A.20}$$

Summing over the G workers writes the estimator (14) as a ratio of worker totals, which defines its population counterpart:

$$\widehat{\lambda}_\ell^{\ell'}(d) = \frac{\sum_{m=1}^G Z_{m,Y}}{\sum_{m=1}^G Z_{m,A}}, \qquad p_\ell^{\ell'}(d) = \frac{\mathbb{E}[Z_{m,Y}]}{\mathbb{E}[Z_{m,A}]}, \tag{A.21}$$

provided the denominators are positive. Write p for this population object. It is an interval exit probability, not the instantaneous hazard $\lambda_\ell^{\ell'}(d)$ of equation (10).

Sampling properties. The estimator is a ratio of two averages across workers, so the law of large numbers and the central limit theorem for independent workers deliver its sampling properties.

Proposition 9 (Consistency and asymptotic normality)**.** *Suppose the worker totals* $(Z_{m,Y}, Z_{m,A})$ *are independent and identically distributed across workers, with* $\mathbb{E}[Z_{m,A}] > 0$ *and finite second moments, while a worker's own spells may be arbitrarily dependent. Then, as* $G \to \infty$*,*

$$\widehat{\lambda}_\ell^{\ell'}(d) \xrightarrow{p} p,$$
$$\sqrt{G}\big(\widehat{\lambda}_\ell^{\ell'}(d) - p\big) \xrightarrow{d} \mathcal{N}\left(0, \frac{\mathrm{Var}(Z_{m,Y} - pZ_{m,A})}{(\mathbb{E}[Z_{m,A}])^2}\right).$$

Proof. Write $\overline{Z}_Y = G^{-1}\sum_m Z_{m,Y}$ and $\overline{Z}_A = G^{-1}\sum_m Z_{m,A}$. The worker-level law of large numbers implies that these averages converge to their expectations. A positive expected number of spells at risk makes the denominator positive with probability approaching one, so their ratio converges to p.

For the limit distribution, center each worker's exits at p times their spells at risk. The estimation error is then an average of these centered contributions, divided by the average number at risk:

$$\sqrt{G}\big(\widehat{\lambda}_\ell^{\ell'}(d) - p\big) = \frac{1}{\overline{Z}_A}\frac{1}{\sqrt{G}}\sum_{m=1}^{G}\big(Z_{m,Y} - pZ_{m,A}\big).$$

The summands have mean zero by the definition of p and finite variance by the second-moment condition. The worker-level central limit theorem gives their normal limit. Slutsky's theorem replaces $\overline{Z}_A$ with $\mathbb{E}[Z_{m,A}]$, yielding the stated variance. □

The same worker-level contributions yield a consistent variance estimator after replacing the population probability with its sample counterpart:

$$\widehat{\mathrm{Var}}\Big(\widehat{\lambda}_\ell^{\ell'}(d)\Big) = \frac{\displaystyle\sum_{m=1}^{G}\big[Z_{m,Y} - \widehat{\lambda}_\ell^{\ell'}(d)Z_{m,A}\big]^2}{\displaystyle\left(\sum_{m=1}^{G} Z_{m,A}\right)^2}. \tag{A.22}$$

Corollary 3 (Worker-clustered variance)**.** *Under the conditions of Proposition 9,*

$$G\,\widehat{\mathrm{Var}}\Big(\widehat{\lambda}_\ell^{\ell'}(d)\Big) \xrightarrow{p} \frac{\mathrm{Var}(Z_{m,Y} - pZ_{m,A})}{(\mathbb{E}[Z_{m,A}])^2}.$$

Proof. It suffices to show that replacing p by its consistent estimate leaves the limit of the average squared contribution unchanged. Expanding the squared fitted contributions produces the average of $(Z_{m,Y} - pZ_{m,A})^2$ and two additional terms containing $\widehat{\lambda}_\ell^{\ell'}(d) - p$ and its square. Finite second moments make the averages of $(Z_{m,Y} - pZ_{m,A})Z_{m,A}$ and $Z_{m,A}^2$ bounded in probability. Consistency therefore makes both additional terms vanish. The law of large numbers gives

$$\frac{1}{G}\sum_{m=1}^{G}\big[Z_{m,Y} - \widehat{\lambda}_\ell^{\ell'}(d)Z_{m,A}\big]^2 \xrightarrow{p} \mathbb{E}[(Z_{m,Y} - pZ_{m,A})^2] = \mathrm{Var}(Z_{m,Y} - pZ_{m,A}).$$

The average worker exposure converges to $\mathbb{E}[Z_{m,A}] > 0$, so division by its square yields the stated limit. Multiplication by a correction factor converging to one preserves this limit. □

A.6 Identification up to Scale

This subsection proves Proposition 1. A change of units relabels time, human capital, and output without changing the economy, much as a rotation or a translation moves a figure without changing its shape. I prove the proposition by showing that, written in the new units, each equation of the model is the original equation multiplied by a positive constant: $c_A c_t$ for each HJB and c_t/c_x for each forward equation. A solution in the old units is then a solution in the new ones. The targeted moments compare objects measured in the same units, so they cannot depend on which units are chosen. Parameter vectors related by a change of units therefore form an equivalence class, and the normalization selects one member of each class.

Table A.1: Units of the model's objects

Object	Measured in	Multiplied by
Rates $\rho, \delta, \delta_i, \delta_f$ and search intensities $\eta_\ell^{\ell'}$	per unit of time	c_t
Drift μ_ℓ	human capital per unit of time	$c_x c_t$
Volatility σ_ℓ	human capital per square root of time	$c_x\sqrt{c_t}$
Values U, V_ℓ and the entry cost Ψ_f	output	c_A
Flow payoffs F, b, wages, output, and transfers	output per unit of time	$c_A c_t$
Search-cost coefficients $\kappa_\ell^{\ell'}$	output times time	c_A/c_t
Production scale A	output per unit of human capital and time	$c_A c_t/c_x$
Human capital, thresholds, and the median k_g	human capital	c_x
Densities m_ℓ and g	per unit of human capital	$1/c_x$

Proof of Proposition 1. Fix positive constants c_t, c_x, and c_A, and write relabeled objects with a tilde. A worker at date t with human capital x has new coordinates $\tilde{t} = t/c_t$ and $\tilde{x} = c_x x$, and a model period lasts $\tilde{a} = c_t a$ months, so $\tilde{a}\tilde{d} = ad$ and calendar durations are unchanged. Every other object is relabeled according to what it measures, and Table A.1 lists the factors. Two of them deserve a word. Volatility scales by $c_x\sqrt{c_t}$ because the variance of a Brownian increment grows linearly with elapsed time. Search-cost coefficients scale by c_A/c_t because a quadratic cost of a rate must still be a flow payoff. For the log-logistic entrant density, relabeling replaces the median k_g by $c_x k_g$, and the dimensionless primitives τ, α_0, and ξ_g are unchanged.

It remains to show that the relabeled objects form an equilibrium of the relabeled economy and generate the same targeted moments. I check each equation of the model in turn, beginning with the law of motion of human capital, on which the others rest. Notice that each step uses nothing but the units in Table A.1.

Consider first the law of motion. That is, the relabeled human capital must follow the law of motion with the drift and volatility of Table A.1. Brownian motion rescales with the clock:

$W(t) = \sqrt{c_t}\,\tilde{W}(\tilde{t})$ for a standard Brownian motion $\tilde{W}$ in the new clock. Hence

$$\mathrm{d}\tilde{x} = c_x\,\mathrm{d}x = c_x c_t \mu_\ell\,\mathrm{d}\tilde{t} + c_x\sqrt{c_t}\,\sigma_\ell\,\mathrm{d}\tilde{W} = \tilde{\mu}_\ell\,\mathrm{d}\tilde{t} + \tilde{\sigma}_\ell\,\mathrm{d}\tilde{W},$$

as required, and reflection at zero is unchanged.

Next, consider the worker problem. That is, the relabeled values must satisfy each HJB with the relabeled primitives. Since the wage wedges τ and α_0 in equation (2) are unchanged, wages scale with the production scale, $\tilde{a}_\ell\tilde{x} = (c_A c_t/c_x)a_\ell \cdot c_x x = c_A c_t w_\ell(x)$, like every other flow payoff. Every term of an HJB then scales by the same factor. For the drift and diffusion terms,

$$\begin{aligned}\tilde{\mu}_\ell\,\partial_{\tilde{x}}\tilde{V}_\ell &= c_x c_t \mu_\ell \cdot \frac{c_A}{c_x}\partial_x V_\ell = c_A c_t\,\mu_\ell \partial_x V_\ell,\\ \frac{\tilde{\sigma}_\ell^2}{2}\partial_{\tilde{x}\tilde{x}}\tilde{V}_\ell &= \frac{c_x^2 c_t \sigma_\ell^2}{2}\cdot\frac{c_A}{c_x^2}\partial_{xx}V_\ell = c_A c_t\,\frac{\sigma_\ell^2}{2}\partial_{xx}V_\ell,\end{aligned}$$

and the discount, separation, and time-derivative terms behave the same way, because rates scale by c_t and values by c_A. So do the search terms, since with $\tilde{\eta} = c_t\eta$,

$$\tilde{\eta}\,c_A\Delta V - \frac{\tilde{\kappa}_\ell^{\ell'}}{2}\tilde{\eta}^2 = c_A c_t\left[\eta\Delta V - \frac{\kappa_\ell^{\ell'}}{2}\eta^2\right],$$

where ΔV includes the applicable fixed entry cost. Dividing each relabeled HJB by $c_A c_t$ recovers the original one, so the relabeled values solve the relabeled worker problem. The first-order condition then gives $\tilde{\eta}_\ell^{\ell'}(\tilde{x},\tilde{t}) = c_t\eta_\ell^{\ell'}(x,t)$, and value matching, smooth pasting, and the formal-entry equality hold with every threshold multiplied by c_x.

With policies in hand, turn to the distribution of workers. That is, the relabeled densities must solve the forward equations with the relabeled policies. The relabeled flux is

$$\begin{aligned}\widetilde{\mathcal{J}}_\ell[\tilde{m}_\ell](\tilde{x},\tilde{t}) &= \tilde{\mu}_\ell\tilde{m}_\ell - \frac{\tilde{\sigma}_\ell^2}{2}\partial_{\tilde{x}}\tilde{m}_\ell\\ &= c_x c_t\mu_\ell\cdot\frac{m_\ell}{c_x} - \frac{c_x^2 c_t\sigma_\ell^2}{2}\cdot\frac{\partial_x m_\ell}{c_x^2} = c_t\mathcal{J}_\ell[m_\ell](x,t).\end{aligned}$$

Every term of each forward equation therefore scales by c_t/c_x: the time derivative, the flux term, each search, separation, and exit flow, the entry flow $\tilde{\delta}\tilde{g} = (c_t/c_x)\delta g$, and the quit flows at thresholds. The relabeled densities solve the relabeled forward equations, and each state holds the same mass of workers at the same calendar date.

It remains to check labor-market clearing and the government budget. Sectoral output scales by $c_A c_t$, since

$$\tilde{A}\int_0^\infty \tilde{x}\,\tilde{m}_\ell(\tilde{x})\,\mathrm{d}\tilde{x} = \frac{c_A c_t}{c_x}A\int_0^\infty c_x x\,m_\ell(x)\,\mathrm{d}x = c_A c_t A\int_0^\infty x\,m_\ell(x)\,\mathrm{d}x.$$

The density mapping preserves labor-market clearing and sectoral population shares. Informal

output losses scale by the same factor as output because α_0 is unchanged. Tax revenue and benefit expenditure in equation (8) both scale by $c_A c_t$, so the budget holds with the relabeled transfer $\widetilde{\mathcal{T}}(\tilde{t}) = c_A c_t \, \mathcal{T}(t)$. The relabeled allocation is therefore an equilibrium.

Finally, consider the targeted moments. Each compares objects measured in the same units, so it cannot depend on which units are chosen. Benefits and output both scale by $c_A c_t$, so $b\mathbf{M}_u/Y$ is unchanged. Completed-spell wages scale like the formal-wage level that normalizes them, so normalized means and variances are unchanged. Hazards scale by c_t, so destination shares and log ratios of hazards at two calendar durations are unchanged. Because $\tilde{a}\tilde{d} = ad$, every profile evaluated at a given number of calendar months is unchanged, and the same mapping of densities and durations preserves the three entry-cohort state shares.

The normalization now follows by choosing the units. Taking

$$c_t = \frac{12}{a}, \qquad c_x = \frac{1}{k_g}, \qquad c_A = \frac{c_x}{c_t A}$$

gives $\tilde{a} = 12$, $\tilde{k}_g = 1$, and $\tilde{A} = 1$, and in particular

$$(\tilde{F}, \tilde{b}) = \frac{c_x}{A}(F, b), \qquad \tilde{\Psi}_f = \frac{c_x}{c_t A}\Psi_f, \qquad \tilde{\kappa}_\ell^{\ell'} = \frac{c_x}{c_t^2 A}\kappa_\ell^{\ell'}.$$

Every parameter vector is thus equivalent to one with $a = 12$, $k_g = 1$, and $A = 1$, the representative the estimation uses. Once a and k_g are fixed, an absolute wage expressed in a fixed empirical output unit determines the remaining output scale A through the inversion in equation (17). □

Note that the argument never solves the model. It only tracks what each object measures, which is why fixing the units costs the estimation nothing.

B Data Appendix

B.1 EPS Sources and Sample Construction

I combine retrospective EPS reports into monthly labor histories, apply sample restrictions, and assess the resulting coverage.

Sources and retrospective reports. The retrospective modules of the 2002, 2004, 2006, 2009, and 2015 EPS waves record work and nonwork periods. Each report gives start and end months, together with employment category, contract and contribution information, earnings, hours, occupation, industry, firm size, job term, and region. Respondents can describe several periods in one interview and revisit periods reported in earlier waves.

Missing a wave therefore need not leave a gap in a worker's history. Table B.1 shows how often histories rely on these later reports. Panel A divides each wave's respondents into first-time, continuous, and returning respondents, and Panel B follows each wave's respondents to later interviews. Panel C measures the same turnover in weighted terms, separating attrition from the inflow of new and returning respondents and from revisions to the expansion factors.

The 2012 wave is missing from every history, because I exclude its substantive responses after interview problems. Its identifiers and expansion factors contribute only to the participation and flow accounting, and the 2015 retrospective module covers the intervening years. Table B.2 assesses how the exclusion affects the histories. Panels A and B follow 2009 respondents forward and 2015 respondents backward across the 2012 wave, and Panel C measures how completely the accepted histories of 2015 respondents cover January 2010 to December 2014. Respondents interviewed in 2012 but not in 2015 have histories that end with their last earlier report, where their final spell is right-censored.

Construction of labor histories. Before pooling these reports, I harmonize identifiers, spell types, job characteristics, and reporting waves and put all dates on a common monthly calendar. I also correct the reversed 2002 work and first-time-search labels and remove reports with invalid month codes, sentinel years, or end dates before their start dates. At this stage, I classify employment within each wave using the rules in Appendix B.2.

To reconcile overlapping and duplicate reports, I score their completeness by counting observed earnings, hours, two firm-size fields, occupation, industry, contract, contributions, region, and job term. A formal or informal work report with at least eight of these ten fields is classified as rich work. In flagged histories, rich work takes overlapping months from nonwork reports with no observed characteristics. I split surrounding records and trim one-sided overlaps, using completeness and rich-work status to resolve remaining conflicts while preserving retained work classifications. For exact same-span duplicates, I give priority to work, then rich formal or informal work, greater completeness, and the reporting wave closest to the start. Each retained piece keeps

Table B.1: EPS respondent composition and subsequent participation

	2002	2004	2006	2009	2015
Panel A. Composition of respondents observed in each wave					
Respondents observed	15,365	14,899	14,856	13,096	14,413
First-time respondents (%)	100.0	7.8	0.0	0.0	13.3
Continuous respondents (%)	0.0	92.2	78.6	66.3	40.8
Returning respondents (%)	0.0	0.0	21.4	33.7	45.9
Panel B. Subsequent status of respondents from each processed origin wave					
Observed next wave (%)	78.4	87.4	78.0	53.7	—
Missed next, returned later (%)	14.4	6.1	6.4	0.0	—
Not observed again (%)	7.2	6.5	15.6	46.3	—

Panel C. Weighted respondent flows Flow	2002–04	2004–06	2006–09
Outflow	21.6	12.4	19.8
First-observed inflow	29.0	0.0	0.0
Return inflow	0.0	12.3	14.0
Weight revision	26.8	14.4	7.0
Net attrition	-34.2	-14.3	-1.2

Notes: Counts are unweighted. Panels A and B use respondent weights. Panel C reports flows in % of the origin-wave stock using raw expansion factors. Negative net attrition denotes growth. The sample includes men and women eligible under Table B.3. Source: EPS 2002–2015, author's calculations.

its original identifiers and dates. This lets me count affected workers and records in Panels B and C of Table B.3, treating a source report as dropped only if none of its pieces remains.

I address errors identified through manual inspection with fixed worker-specific rules, coded to run in the prescribed record order. These rules recover fields, remove duplicated or implausible reports, adjust boundaries, split records, and remove isolated formality changes attributed to reporting errors. In making these corrections, I close only specified local gaps through boundary adjustments and leave other unreported months unassigned, excluding histories that cannot be reconciled.

Once reports have been reconciled, I consolidate those with matching spell types and employment states when they overlap or meet the survey-boundary condition, and combine consecutive unemployment and other non-employment reports. Each consolidated spell retains the first start, last end, contributing waves, modal categorical characteristics, and mean reported earnings and hours. A formal–informal change always starts a new spell, and the final histories must consist of positive-duration, nonoverlapping spells. Subsequent sector-spell linkage and censoring follow the rules in Appendix B.2.

Analysis sample. The cleaned-history sample includes workers with observed sex and birth year, at least one formal or informal employment spell, and a positive weight from the first processed

Table B.2: 2012 survey participation and retrospective labor-history coverage

Classification	Workers	Unweighted (%)	Weighted (%)
Panel A. Subsequent participation of 2009 respondents			
Neither 2012 nor 2015	2,895	22.1	30.6
2012 only	2,124	16.2	15.7
2015 only	2,316	17.7	18.4
Both 2012 and 2015	5,761	44.0	35.3
Panel B. Prior participation of 2015 respondents			
Neither 2009 nor 2012	5,928	41.1	54.5
2009 only	2,316	16.1	16.4
2012 only	408	2.8	4.7
Both 2009 and 2012	5,761	40.0	24.4

Panel C. Coverage of 2010–2014 in accepted histories of 2015 respondents	Workers	Mean coverage (%)	Complete coverage (%)	Mean months last reported in 2015
Observed in 2009 and 2015	8,077	100.0	99.9	60.0
Earlier returner, absent in 2009	1,172	100.0	100.0	60.0
First processed in 2015	5,164	96.2	83.4	57.7

Notes: Counts and coverage statistics are unweighted. Participation percentages use 2009 respondent weights in Panel A and 2015 weights in Panel B. Coverage concerns January 2010–December 2014 among 2015 respondents of both sexes. Complete coverage requires all 60 months. In Panel C, the last column averages across respondents the months of this window that fall in spells last reported in 2015. Source: EPS 2002–2015, author's calculations.

interview. Applying these restrictions after excluding unreliable histories leaves 23,205 men and women, as reported in Panel A of Table B.3. Because reports can be removed or consolidated without excluding their workers, the record counts in Panels B and C are reported separately.

The duration and mobility analyses then restrict these histories to men ages 15–64, whereas Tables B.1–B.4 cover the broader cleaned-history sample. Additional wage-specific observation rules are given in Appendix B.3, with weighting described in Appendix B.2.

Table B.3: EPS sample construction and labor-history cleaning

Panel A. Worker-level sample construction

Sample stage	Workers retained	Workers removed	Share of initial workers (%)
Enter spell-history cleaning	23,321	—	100.00
After unreliable-history exclusions	23,315	6	99.97
Valid person record and baseline weight	23,315	0	99.97
At least one formal or informal spell	23,205	110	99.50
Cleaned-history sample	23,205	0	99.50

Panel B. Spell-record cleaning and boundary adjustments

Cleaning operation	Records affected	Workers affected	Record sample remaining (%)
Spell records entering cleaning	192,990	23,321	100.00
Source spell records dropped	756	273	99.61
Records created by splitting	44	40	99.63
Boundaries moved to resolve overlaps	426	220	99.63
Boundaries moved to close gaps	6	6	99.63
Spell records after cleaning	192,278	23,315	99.63

Panel C. Consolidation of cleaned records into final spells

Cleaning operation	Records affected	Workers affected	Record sample remaining (%)
Date inconsistencies removed	320	316	99.47
Short overlaps removed	907	766	99.00
Compatible records combined	23,485	10,611	86.83
Additional records removed	14	14	86.82
Adjacent non-employment combined	1,500	1,290	86.04
Same-span duplicates removed	18	18	86.03

Cleaned-history sample: 165,696 cleaned spells

Notes: Unweighted worker and record counts for men and women. Percentages use initial workers in Panel A and initial source records in Panels B and C. Panel B's operation counts may overlap. Source: EPS 2002–2015, author's calculations.

Coverage and construction checks. Table B.4 checks the continuity and depth of the cleaned histories. It counts the gaps longer than one month that remain between adjacent spells and summarizes the years and spells each history covers. A history counts as complete when no gap exceeds one month, but its observed time still excludes every uncovered month, including gaps of one month or less.

Table B.4: Continuity and depth of cleaned EPS labor histories

Panel A. Continuity of cleaned labor histories

Diagnostic	Events	Workers	Share of workers (%)
Workers in the cleaned-history sample	—	23,205	100.0
Gaps longer than one month	1,302	1,161	5.0
Complete histories (gaps of 0–1 month)	—	22,044	95.0
Overlapping adjacent spells	0	0	0.0

Panel B. Depth of cleaned labor histories across workers

History characteristic	Mean	25th percentile	Median	75th percentile
Time covered by spells (years)	24.0	14.2	26.4	34.3
Cleaned spells per worker	7.1	4.0	6.0	9.0
Gap length above one month	17.9	8.0	12.0	12.0

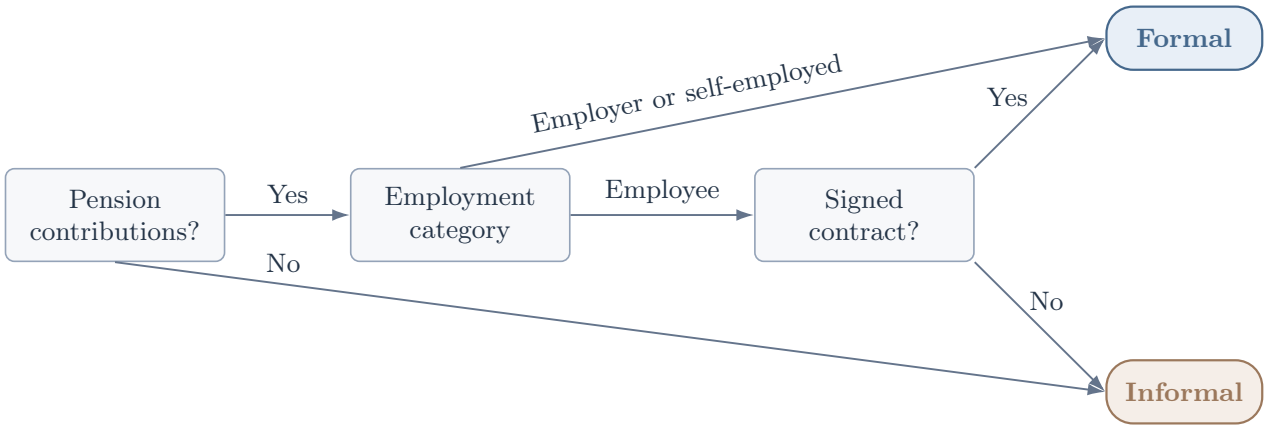
Notes: Unweighted statistics for men and women. Complete histories allow at most one uncovered month between adjacent spells. Gap-length statistics condition on longer gaps. Observed years exclude uncovered months. Source: EPS 2002–2015, author's calculations.

B.2 Labor-Market States and Sector-Spells

Employment classification. The EPS questions on pension contributions, employment category, and signed contracts supply the inputs to the baseline classification in Figure B.1. Contribution responses 1–6 indicate contributions to a pension system and 7 explicit noncontribution. Employment categories 1 and 2 identify employers and the self-employed, whose classification uses contributions alone. For employees, contract codes 1–3 indicate a signed contract and 4 its absence in 2002, whereas later waves use 1 and 2–3, respectively. Unknown responses supply no evidence of noncompliance, although an employee record is informal whenever either requirement is explicitly absent, even if the other is unreported. Other unresolved working records remain unclassified, while nonwork and first-time search records are assigned to unemployment. Unpaid family work and other special employment categories, including Army or police records, remain in the history but are excluded from the formal–informal comparison.

Sector-spell construction. I join consecutive same-state records when the next start month is no more than six months after the previous end, keeping armed-forces or unclassifiable occupation records (code 10) separate. A state change or a longer gap starts a new sector-spell. Because dates

Figure B.1: Classification of employment spells



Regular employment spells with explicit yes/no responses.

are inclusive, a record ending in January and one beginning in July meet this condition despite five uncovered months between them. These months enter elapsed sector-spell duration, measured from the first start through the last end, whereas the observed-time coverage in Appendix B.1 counts only months assigned to records.

The next sector-spell identifies an exit destination only when its start is within six months of the origin's end and it satisfies the destination restrictions. I construct these links from the full chronology before excluding records by age or occupation, so an excluded record cannot be bypassed to create a link to a later spell. Age is evaluated at the start of each constituent job record, and a sector-spell is age-eligible if at least one starts between ages 15 and 64. The mobility statistics then use only links between formal, informal, and unemployed states.

Observing any subsequent sector-spell marks the origin duration as completed, including when a long gap or destination exclusion prevents recording a usable exit. I treat the final observed spell as right-censored at the end of its recorded history. At the other end of the window, spells beginning in January 1980 are potentially left-censored, as flagged in Table B.5. They remain in the baseline, with the exclusion exercise reported in Figure D.1.

Table B.5: Sector-spell construction and censoring diagnostics

	Formal	Informal	Unemployment
All sector-spells	24,115	13,633	23,506
Completed sector-spells	17,978	11,153	20,982
Potentially left-censored January 1980 starts	3,456	1,380	772

Notes: Unweighted sector-spell counts for men satisfying the age and occupation restrictions above.
Source: EPS 2002–2015, author's calculations.

Respondent weights. Within the labor-history sample, I rescale the EPS expansion factors by their mean across observed respondent-wave records in the same entry cohort. The cohort is defined

by the first processed interview, and its normalization uses all available waves. For interviews after entry, I also multiply the rescaled factor by the inverse predicted probability of response. A weighted logit models response using the target and previous observed waves, years since that observation, entry cohort, and previously observed sex, age, education, and region. Respondents may miss an interview and return later. Entry interviews receive no response adjustment, while predicted probabilities for later interviews are bounded between 0.01 and 0.99. The resulting weights are relative analysis weights.

Spell and month weights. Each cleaned job spell inherits the respondent weight from its last contributing reporting wave. For duration and mobility statistics, I assign each sector-spell the duration-weighted average of its component job-spell weights. Component durations count only recorded months, and the resulting flow weight applies once per sector-spell. For stock statistics, I multiply each job spell's weight by its observed duration to obtain its spell-month weight. When expanding histories to one row per month, I divide this weight by the spell's length, so each observed month carries its reporting-wave weight once.

B.3 Wage Measurement

Wage reports and units. I construct a standardized monthly wage equivalent from reported monthly net labor income and weekly hours. Income values at or below 10 CLP or at or above 100 million CLP are treated as missing, as are weekly hours outside 0–168. Before consolidating repeated reports, I deflate each income report to 2010 Chilean pesos with its reporting wave's annual CPI and convert it to U.S. dollars at the 2010 annual-average market exchange rate. This assumes income is reported in contemporaneous nominal units. I then average real income and hours across repeated reports of the same job. For positive income and hours, the hourly wage divides real monthly income by 4.35 times weekly hours. Multiplying it by 40×4.35 gives the monthly equivalent for a common 40-hour week. The 2002 wave supplies no income report, although a later report can supply wage information for a consolidated job that includes 2002.

Wage spells and averages. I combine contiguous job records in the same employment state into a wage sector-spell, allowing employer changes within that state. An uncovered month starts a new wage spell, unlike the six-month link used for mobility in Appendix B.2. Every component job must have a valid positive wage. The wage sample contains men ages 15–64 at the start of the wage sector-spell and excludes the entire spell if any component has occupation code 10 (armed forces or unclassifiable occupations). For a spell in sector ℓ, let w_h be component job h's monthly wage equivalent and L_h its observed duration in months. The arithmetic average corresponding to $\overline{w}_\ell$ in Section 3.3 is

$$\overline{w}_\ell = \frac{\sum_h L_h w_h}{T}, \qquad T = \sum_h L_h.$$

This assigns each component wage to its recorded months without recovering the within-job wage path or the wage at entry. Survey weights apply when averaging across spells, as described in Appendix B.2, and do not enter this within-spell time average.

B.4 First-Time Job-Search Cohort

I identify labor-market entrants whose recorded histories begin with an explicit report of first-time job search. I mark this report before consolidation and preserve the marker when search records merge with other nonemployment records. Entry is dated from the start of the resulting initial nonemployment spell, rather than the first EPS interview. The cohort contains men from the labor-history sample in Appendix B.1, which requires at least one report of regular work somewhere in the history. No additional entry-age restriction is applied.

I follow these entrants for five years, reconstructing their formal, informal, and nonemployment states from the recorded spells. A worker remains in the follow-up sample if every six-month interval contains some observed time in one of these states. Months without an observed state contribute neither a state nor exposure. I hold each worker's survey weight fixed at the weight assigned to the entry spell and pool observed person-months with these weights to calculate the six-month state shares. The coverage requirement retains 122 of 130 male entrants. Among those retained, recorded states cover 99.97 percent of potential person-month exposure across the 61 monthly observations from entry through month 60, using the fixed entry weights.

C Estimation and Numerical Methods

C.1 Solving the Stationary Equilibrium

Wages in Equation (2) depend only on model parameters. I therefore compute the stationary equilibrium in sequence, from workers' optimal choices to the stationary distribution and aggregate outcomes.

Equilibrium algorithm. At each parameter vector, the calculation proceeds as follows:

1. Compute formal and informal wages from Equation (2), using the normalization $A = 1$.
2. Use Howard policy iteration to solve the worker problem at these wages, obtaining U, V_i, and V_f, the search policies in Equation (5), and the employment continuation regions.
3. Solve the stationary KFE under these policies and normalize the worker population to one.
4. Aggregate the stationary densities to obtain employment shares and output.

Numerical setup. For each equilibrium calculation, I fix the upper grid boundary $x_{\max}$, the number of nodes N_x, and the numerical tolerance ϵ. Set

$$\Delta x = \frac{x_{\max}}{N_x - 1}, \qquad x_j = (j-1)\Delta x, \qquad j = 1, \dots, N_x.$$

Let Id be the $N_x \times N_x$ identity, $\mathbf{1}$ the vector of ones, and $(\mathbf{w}_\ell)_j = w_\ell(x_j)$ the wage vectors. These objects remain fixed during the equilibrium calculation.

The tridiagonal matrix $A_\ell = (a^\ell_{jk})$ approximates the human-capital generator. Central differences for diffusion and an upwind difference for drift give, at an interior node,

$$a^\ell_{jk} = \begin{cases} \dfrac{\sigma^2_\ell}{2(\Delta x)^2} + \dfrac{[\mu_\ell]^+}{\Delta x}, & k = j+1, \\ \dfrac{\sigma^2_\ell}{2(\Delta x)^2} + \dfrac{[-\mu_\ell]^+}{\Delta x}, & k = j-1, \\ -a^\ell_{j,j-1} - a^\ell_{j,j+1}, & k = j, \\ 0, & \text{otherwise.} \end{cases}$$

Thus $(\mathcal{L}_\ell V_\ell)(x_j)$ is approximated by $(A_\ell \mathbf{V}_\ell)_j = \sum_k a^\ell_{jk} V_{\ell,k}$.

At the lower endpoint $x_1 = 0$, the only nonzero entries in the first row are

$$a^\ell_{1,2} = \frac{\sigma^2_\ell}{2(\Delta x)^2} + \frac{[\mu_\ell]^+}{\Delta x}, \qquad a^\ell_{1,1} = -a^\ell_{1,2}.$$

At the upper endpoint $x_{N_x} = x_{\max}$, the last row has

$$a^{\ell}_{N_x,N_x-1} = \frac{\sigma_\ell^2}{2(\Delta x)^2} + \frac{[-\mu_\ell]^+}{\Delta x}, \qquad a^{\ell}_{N_x,N_x} = -a^{\ell}_{N_x,N_x-1},$$

with all other entries zero. These rows prevent transitions outside the grid and preserve zero row sums, imposing reflection at both endpoints. Employment quitting is imposed separately below.

For policy evaluation, I also precompute

$$B_u = (\rho + \delta)\mathrm{Id} - A_u, \qquad B_\ell = (\rho + \delta + \delta_\ell)\mathrm{Id} - A_\ell, \quad \ell \in \{i, f\}.$$

Value functions and optimal policies. At iteration n, the grid values $U^n_j = U^n(x_j)$ and $V^n_{\ell,j} = V^n_\ell(x_j)$ form the column vectors $\mathbf{U}^n$ and $\mathbf{V}^n_\ell$. I stack them as

$$\mathbf{v}^n = \begin{bmatrix} \mathbf{U}^n \\ \mathbf{V}^n_i \\ \mathbf{V}^n_f \end{bmatrix}.$$

I solve for $\mathbf{v}$ using Howard policy iteration:

1. *Initialize values.* Choose $\mathbf{v}^0$ and set $n = 0$.

2. *Update search policies.* Evaluate the optimal search rules in Equation (5) at $\mathbf{v}^n$. Let $\mathsf{S}^{\ell'}_\ell$ be the diagonal matrix of search intensities from state ℓ to state ℓ', and let $\mathbf{c}_\ell$ contain the associated quadratic search costs. Holding these policies fixed gives the continuation system

$$B^n = \begin{bmatrix} B_u + \mathsf{S}^i_u + \mathsf{S}^f_u & -\mathsf{S}^i_u & -\mathsf{S}^f_u \\ -\delta_i \mathrm{Id} & B_i + \mathsf{S}^f_i & -\mathsf{S}^f_i \\ -\delta_f \mathrm{Id} & -\mathsf{S}^i_f & B_f + \mathsf{S}^i_f \end{bmatrix},$$

with payoff vector

$$\mathbf{r}^n = \begin{bmatrix} b\mathbf{1} - \mathbf{c}_u - \Psi_f \mathsf{S}^f_u \mathbf{1} \\ \mathbf{w}_i - \mathbf{c}_i - \Psi_f \mathsf{S}^f_i \mathbf{1} \\ \mathbf{w}_f - F\mathbf{1} - \mathbf{c}_f \end{bmatrix}.$$

The Ψ_f terms charge the formal-entry cost at the corresponding arrival rates.

3. *Update quitting decisions.* Compute the continuation residual

$$\mathbf{R}^n = B^n \mathbf{v}^n - \mathbf{r}^n.$$

At employment node j, select quitting when

$$V^n_{\ell,j} - U^n_j < R^n_{\ell,j}.$$

At each such node, replace the employment equation by

$$V_{\ell,j}^{n+1} - U_j^{n+1} = 0.$$

The replacement row has coefficient 1 on the employment value, coefficient -1 on the unemployment value at the same node, and zero elsewhere. Its right-hand side is zero. Retain the continuation equation at all other employment nodes, and denote the resulting system by $\widetilde{B}^n$ and $\widetilde{\mathbf{r}}^n$.

4. *Evaluate policies and update values.* Solve the sparse linear system

$$\widetilde{B}^n \mathbf{v}^{n+1} = \widetilde{\mathbf{r}}^n.$$

Assign the solution blocks as

$$\begin{aligned} \mathbf{U}^{n+1} &= \mathbf{v}_{1:N_x}^{n+1}, \\ \mathbf{V}_i^{n+1} &= \mathbf{v}_{N_x+1:2N_x}^{n+1}, \\ \mathbf{V}_f^{n+1} &= \mathbf{v}_{2N_x+1:3N_x}^{n+1}. \end{aligned}$$

This updates all three value functions jointly. At quitting nodes, the new employment value equals the new unemployment value.

5. *Recompute the optimized residuals.* Evaluate search intensities and costs at $\mathbf{v}^{n+1}$, then reconstruct the continuation matrix B^{n+1} and payoff vector $\mathbf{r}^{n+1}$ from step 2. Before replacing any quitting rows, compute

$$\mathbf{R}^{n+1} = B^{n+1}\mathbf{v}^{n+1} - \mathbf{r}^{n+1}.$$

Its three blocks are $\mathbf{R}_u^{n+1}$, $\mathbf{R}_i^{n+1}$, and $\mathbf{R}_f^{n+1}$.

6. *Check convergence.* With minima taken entry by entry, check whether

$$\left\| \begin{bmatrix} \mathbf{R}_u^{n+1} \\ \min\{\mathbf{R}_i^{n+1}, \mathbf{V}_i^{n+1} - \mathbf{U}^{n+1}\} \\ \min\{\mathbf{R}_f^{n+1}, \mathbf{V}_f^{n+1} - \mathbf{U}^{n+1}\} \end{bmatrix} \right\|_\infty \leq \epsilon.$$

If this condition holds, accept the value functions and end the loop. Otherwise, set $n \leftarrow n+1$ and return to step 2.

At the solution, I locate each quitting threshold using the employment surplus. Let $\bar{j}_\ell$ be the index immediately below the first grid node with strictly positive surplus. When $\bar{j}_\ell \geq 1$,

$$V_{\ell,\bar{j}_\ell} - U_{\bar{j}_\ell} \simeq 0, \qquad V_{\ell,\bar{j}_\ell+1} - U_{\bar{j}_\ell+1} > 0.$$

The last indifferent grid point, $x_{\bar{j}_\ell}$, approximates the threshold $\bar{x}_u^\ell$. If employment continues at the first grid point, set $\bar{j}_\ell = 0$ and retain reflection at zero.

Stationary distribution. Given the converged search policies and quitting indices $\bar{j}_i$ and $\bar{j}_f$, I solve the stationary KFE on the same human-capital grid. Let $m_\ell = (m_\ell(x_1), \ldots, m_\ell(x_{N_x}))'$ denote the density vector in state ℓ, and stack the unknowns as

$$\mathbf{m} = \begin{bmatrix} m_u \\ m_i \\ m_f \end{bmatrix}.$$

Let $\mathsf{S}_\ell^{\ell'}$ denote the diagonal matrix of converged search intensities,

$$\mathsf{S}_\ell^{\ell'} = \operatorname{diag}\Big(\eta_\ell^{\ell'}(x_1), \ldots, \eta_\ell^{\ell'}(x_{N_x})\Big).$$

These matrices remain fixed throughout the stationary-distribution calculation.

The calculation proceeds as follows:

1. *Form the forward spatial operators.* The transpose A_ℓ' maps the spatial transition rates from the worker problem into density inflows and outflows. Initialize the joint operator as

$$G_0 = \begin{bmatrix} A_u' & 0 & 0 \\ 0 & A_i' & 0 \\ 0 & 0 & A_f' \end{bmatrix}.$$

   Within each block, columns index origins and rows index destinations. Write $G_{0,\ell\ell'}$ for the block mapping density from state ℓ' into state ℓ. At this stage, the spatial operators retain the reflecting endpoints defined above.

2. *Impose quitting and transfer the boundary flow.* For each employment state, density is zero at quitting nodes:

$$m_\ell(x_j) = 0 \quad \text{for } j \le \bar{j}_\ell, \qquad \ell \in \{i, f\}.$$

   At a positive quitting threshold, employment ends when human capital first reaches the boundary. The continuous boundary condition and quit flow satisfy

$$\lim_{x \downarrow \bar{x}_u^\ell} m_\ell(x) = 0,$$

$$Q_\ell = -\lim_{x \downarrow \bar{x}_u^\ell} \mathcal{J}_\ell[m_\ell](x) = \frac{\sigma_\ell^2}{2} \lim_{x \downarrow \bar{x}_u^\ell} m_\ell'(x).$$

   Thus zero boundary density can coexist with a positive flow into unemployment.

   To implement this boundary, zero the rows and columns of $G_{0,\ell\ell}$ corresponding to $j \le \bar{j}_\ell$. Retain the original diagonal at the first continuation node, $\bar{j}_\ell + 1$. The downward transition rate $a^\ell_{\bar{j}_\ell+1,\bar{j}_\ell}$ then remains a loss from employment.

   For $\bar{j}_\ell \ge 1$, let e_j denote the jth coordinate vector and define

$$\mathsf{T}_\ell = a^\ell_{\bar{j}_\ell+1,\bar{j}_\ell} e_{\bar{j}_\ell} e'_{\bar{j}_\ell+1}.$$

Transfer the lost density into unemployment by updating

$$G_{0,u\ell} \leftarrow G_{0,u\ell} + \mathsf{T}_\ell.$$

The associated aggregate quit flow is

$$Q_\ell^{\Delta x} = \Delta x \, a^\ell_{\bar{j}_\ell+1,\bar{j}_\ell} m_\ell(x_{\bar{j}_\ell+1}).$$

Employment loses this flow at its first continuation node, and unemployment receives it at the adjacent node. If $\bar{j}_\ell = 0$, the lower boundary reflects and $\mathsf{T}_\ell = 0$. The upper endpoint retains reflection in either case.

3. *Add search and exogenous separations.* Update

$$G_0 \leftarrow G_0 + \begin{bmatrix} -\mathsf{S}_u^i - \mathsf{S}_u^f & \delta_i \mathrm{Id} & \delta_f \mathrm{Id} \\ \mathsf{S}_u^i & -\mathsf{S}_i^f - \delta_i \mathrm{Id} & \mathsf{S}_f^i \\ \mathsf{S}_u^f & \mathsf{S}_i^f & -\mathsf{S}_f^i - \delta_f \mathrm{Id} \end{bmatrix}.$$

These transitions preserve human capital. Each departure therefore appears as a loss from the origin state and an equal inflow into the destination state at the same grid point. Together with the quitting transfers, they preserve worker mass.

4. *Add replacement entry and solve for stationary densities.* Normalize the entrant density using

$$\Delta x \sum_{j=1}^{N_x} g(x_j) = 1, \qquad \mathbf{g} = \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix}.$$

Including labor-force exit and replacement gives the full transition operator

$$G = G_0 - \delta \mathrm{Id}_{3N_x} + \delta \mathbf{g} \, \Delta x \, \mathbf{1}',$$

where Id_{3N_x} is the $3N_x \times 3N_x$ identity matrix. The stationary density satisfies

$$G\mathbf{m} = 0, \qquad \Delta x \, \mathbf{1}'\mathbf{m} = 1.$$

Under unit population mass, replacement entry can be moved to the right-hand side, yielding the sparse system

$$(G_0 - \delta \mathrm{Id}_{3N_x})\mathbf{m} = -\delta \mathbf{g}.$$

Replace each inactive employment row by $m_\ell(x_j) = 0$ and the first unemployment row by the population normalization. Solve the resulting system jointly for m_u, m_i, and m_f. Entry

normalization, stationary masses, and aggregate output use rectangular quadrature with weight Δx at every node.

5. *Verify the solution.* Evaluate the stationary residual using the full operator G, before any rows are replaced to impose constraints. Accept the density when

$$\|G\mathbf{m}\|_\infty \leq \epsilon,$$

with unit population mass and nonnegative densities up to numerical tolerance.

C.2 Numerical Calculation of Model Moments

I calculate model moments using the equilibrium densities, policies, and transition operators obtained in Appendix C.1, retaining the same human-capital grid and rectangular quadrature. I first compute stationary shares, then use spell and entrant cohorts to calculate dynamic moments and completed-spell wage means.

Stationary shares. Summing the stationary densities gives the population shares in each labor-market state,

$$\mathbf{M}_u = \Delta x \sum_{j=1}^{N_x} m_u(x_j), \qquad \mathbf{M}_\ell = \Delta x \sum_{j=\bar{j}_\ell+1}^{N_x} m_\ell(x_j), \quad \ell \in \{i, f\}.$$

Since total population is normalized to one, these shares sum to one. Informality as a share of employment is $\mathbf{M}_i/(\mathbf{M}_i + \mathbf{M}_f)$.

Initial cohorts and numerical propagation. Dynamic moments follow either workers beginning a new spell or workers entering the labor force. I construct their initial distributions and propagate them using the fixed equilibrium operators from C.1.

1. *Construct the entrant distribution.* With median entrant human capital normalized to one, evaluate the log-logistic density on the grid and normalize its numerical mass:

$$\widetilde{g}_j = \frac{\xi_g x_j^{\xi_g - 1}}{(1 + x_j^{\xi_g})^2}, \qquad g_j = \frac{\widetilde{g}_j}{\Delta x \sum_{r=1}^{N_x} \widetilde{g}_r}, \qquad g = (g_1, \ldots, g_{N_x})'.$$

Since $\xi_g > 1$, the density at $x_1 = 0$ is zero. New workers enter unemployment, so their initial joint density is

$$z^0 = \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix}.$$

2. *Construct the spell-entry distributions.* New spells begin with the stationary flows into each state. Using the search and quitting matrices from C.1, these inflow densities are

$$\begin{aligned}\mathbf{b}_u &= (\delta_i \mathrm{Id} + \mathsf{T}_i)m_i + (\delta_f \mathrm{Id} + \mathsf{T}_f)m_f + \delta g,\\ \mathbf{b}_i &= \mathsf{S}_u^i m_u + \mathsf{S}_f^i m_f,\\ \mathbf{b}_f &= \mathsf{S}_u^f m_u + \mathsf{S}_i^f m_i.\end{aligned}$$

Unemployment spells can therefore begin through separation, quitting, or labor-force entry. Normalize each inflow to obtain the density of workers beginning a spell:

$$\phi_\ell^0 = \frac{\mathbf{b}_\ell}{\Delta x\, \mathbf{1}'\mathbf{b}_\ell}, \qquad \ell \in \{u, i, f\}.$$

3. *Select the cohort operator.* For a spell cohort, the first exit ends the spell. Its operator is therefore the corresponding diagonal block of G_0. For unemployment,

$$G_{0,uu} = A_u' - \mathsf{S}_u^i - \mathsf{S}_u^f.$$

At employment continuation nodes, the entries are

$$\begin{aligned}(G_{0,ii})_{jr} &= a_{rj}^i - (\mathsf{S}_i^f)_{jr} - \delta_i \mathbf{1}\{j = r\}, \quad j, r > \bar{j}_i,\\ (G_{0,ff})_{jr} &= a_{rj}^f - (\mathsf{S}_f^i)_{jr} - \delta_f \mathbf{1}\{j = r\}, \quad j, r > \bar{j}_f.\end{aligned}$$

The quitting adjustments imposed in C.1 retain the boundary loss, and inactive employment nodes carry zero density. Without labor-force exit, set

$$K_\ell^0 = G_{0,\ell\ell}.$$

Including labor-force exit gives $K_\ell^\delta = K_\ell^0 - \delta \mathrm{Id}$. I use K_ℓ^0 for discrete hazards and destination shares, and K_ℓ^δ for completed-spell wage means.

The entrant cohort instead follows the full operator

$$G_0 - \delta \mathrm{Id}_{3N_x},$$

allowing workers to change states repeatedly. Replacement births are excluded so that the calculation continues to follow the original entrants.

4. *Propagate the cohorts.* Let K_ℓ denote the chosen spell operator. With time step h, update

$$\phi_\ell^{n+1} = (\mathrm{Id} + hK_\ell)\phi_\ell^n, \qquad S_\ell^n = \Delta x\, \mathbf{1}'\phi_\ell^n.$$

Choose h so that the transition matrix is nonnegative. Repeat until the required horizon is reached or the remaining mass falls below tolerance ϵ.

For the entrant cohort, update the joint density using

$$z^{n+1} = z^n + h_n(G_0 - \delta \mathrm{Id}_{3N_x})z^n.$$

Choose an admissible nominal step and shorten the final step to each requested observation date so that the numerical path reaches that date exactly.

Cohort shares and discrete hazards. The propagated entrant and spell cohorts give model counterparts over the same observation intervals as the data. Calendar durations are converted to model time using the convention in Appendix C.3.

1. *Calculate entrant state shares.* From the joint entrant density $z(t)$, calculate the share in state ℓ among workers who remain in the labor force:

$$\frac{\Delta x\, \mathbf{1}' z_\ell(t)}{\Delta x\, \mathbf{1}' z(t)}.$$

Evaluate these shares at monthly observation dates and average them within each reported six-month interval using the empirical monthly exposure weights. The first interval includes the initial observation at month zero.

2. *Calculate discrete hazards.* For a reporting interval ending at d with width Δ, let n_0 and n_1 be the numerical indices at its beginning and end. Using the survivor mass already calculated in the propagation step,

$$\lambda_\ell^{\ell'}(d;\Delta) = \frac{h\,\Delta x}{S_\ell^{n_0}} \sum_{n=n_0}^{n_1-1} \mathbf{1}' G_{0,\ell'\ell} \phi_\ell^n.$$

The numerator sums exits to ℓ' over the interval, and the denominator is the cohort mass at risk at its start. The block $G_{0,\ell'\ell}$ includes the relevant search flows and, for moves into unemployment, separations and quits. I use six-month intervals for the discrete hazard counterparts, with the numerical step chosen to divide the interval width. When labor-force exit is included, it reduces survival but is excluded from the labor-market exit numerator.

3. *Calculate destination shares.* Calculate the discrete hazards over eighteen-month intervals, then take

$$\frac{\lambda_\ell^{\ell'}(d;\Delta)}{\sum_{r\neq\ell} \lambda_\ell^r(d;\Delta)}.$$

The common initial risk-set denominator cancels, giving the share of exits to ℓ' within the interval.

The interval argument distinguishes the discrete hazard from the instantaneous hazard $\lambda_\ell^{\ell'}(t)$ defined in the model. The connection follows by integrating the instantaneous hazard over the interval. If the initial risk set is representative and each included spell is observed until its first

exit or the interval's end, their population relationship is

$$\lambda_\ell^{\ell'}(d;\Delta) = \int_{d-\Delta}^{d} \frac{S_\ell(t)}{S_\ell(d-\Delta)} \lambda_\ell^{\ell'}(t)\,\mathrm{d}t. \tag{C.1}$$

Survival and instantaneous hazards refer to the same cohort and competing-exit convention. For a short interval with little change in survival and the hazard, $\lambda_\ell^{\ell'}(d;\Delta)/\Delta$ approximates the instantaneous hazard.

For destinations r and s with $\lambda_\ell^s(t) > 0$ throughout the interval,

$$\frac{\lambda_\ell^r(d;\Delta)}{\lambda_\ell^s(d;\Delta)} = \frac{\displaystyle\int_{d-\Delta}^{d} S_\ell(t)\lambda_\ell^s(t)\frac{\lambda_\ell^r(t)}{\lambda_\ell^s(t)}\,\mathrm{d}t}{\displaystyle\int_{d-\Delta}^{d} S_\ell(t)\lambda_\ell^s(t)\,\mathrm{d}t}.$$

The discrete relative hazard is a positive weighted average of the instantaneous relative hazard. It lies between its infimum and supremum within the interval and preserves its ordering across nonoverlapping intervals when the instantaneous ratio is monotone. With representative initial risk sets, this ordering also allows incomplete observation if, conditional on exit duration, the probability of recording the exit and its destination is common across destinations. Recording reweights the average wherever its denominator is positive, although individual discrete hazards need not equal their fully observed counterparts.

Completed-spell wage means. For each employment state $\ell \in \{i, f\}$, I calculate the mean of completed-spell average wages, weighting each completed spell by its duration. The calculation uses the initial spell distribution ϕ_ℓ^0, the operator K_ℓ^δ, and the wage vector $\mathbf{w}_\ell$. Moves to another labor-market state complete the spell, while labor-force exit censors it.

1. *Form the spell transition matrix.* Using an admissible time step h, set

$$P_\ell = \mathrm{Id} + hK_\ell^\delta.$$

   Inactive employment nodes retain zero density.

2. *Calculate time spent at each human-capital node.* Starting from a unit mass of new spells, add the surviving cohort densities over time:

$$\mathbf{o}_\ell = h\sum_{n=0}^{\infty} \phi_\ell^n.$$

   This occupation density records where workers spend their time before leaving the state or the labor force. In particular, $\Delta x\, o_{\ell,j}$ is the expected time spent in grid cell j. The recursive

cohort law allows this infinite sum to be calculated through the sparse system

$$-K_\ell^\delta \mathbf{o}_\ell = \phi_\ell^0.$$

3. *Calculate duration and accumulated wage weights.* To weight completed spells by duration, I sum the surviving density weighted by elapsed time. To construct their wages, I also sum the surviving density weighted by wage income accumulated during the spell. Denote these two vectors by $\mathbf{d}_\ell$ and $\mathbf{r}_\ell$, respectively. They satisfy

$$\begin{aligned} -K_\ell^\delta \mathbf{d}_\ell &= P_\ell \mathbf{o}_\ell, \\ -K_\ell^\delta \mathbf{r}_\ell &= P_\ell(\mathbf{w}_\ell \odot \mathbf{o}_\ell). \end{aligned}$$

   The factors P_ℓ preserve evaluation at positive right endpoints of the numerical steps. All three systems reuse one factorization of $-K_\ell^\delta$ and sum the full numerical spell horizon without truncating its tail.

4. *Aggregate completed spells.* Apply the existing completion blocks to the wage and duration vectors:

$$L_{1,\ell}^M = \frac{\sum_{\ell' \neq \ell} \mathbf{1}' G_{0,\ell'\ell} \mathbf{r}_\ell}{\sum_{\ell' \neq \ell} \mathbf{1}' G_{0,\ell'\ell} \mathbf{d}_\ell}.$$

   The numerator pools accumulated wage income at completion, and the denominator pools completed-spell durations. The common quadrature factor Δx cancels. The completion blocks include direct employment moves, separations, and quitting. Labor-force exit affects the surviving densities through K_ℓ^δ but contributes to neither completion total.

I check the linear-system residuals and nonnegativity within numerical tolerance ϵ. Wage normalization and conversion to dollar units are described in Appendix C.3.

C.3 External Calibration and Normalizations

I set the external parameters and labor-force exit rate before solving the model, convert the inputs to the model clock, and recover dollar units from the formal wage level after estimation.

External parameters. I calibrate ρ using annual average yields on five- and ten-year BCU bonds issued by the Banco Central de Chile. These bonds are denominated in the inflation-indexed Unidad de Fomento (Banco Central de Chile, 2026). Let $r_{m,y}^{\text{BCU}}$ denote the annual yield in decimal form for maturity m and year y. Treating each yield as an effective annual real return, I

convert it to a continuously compounded rate and give each year and maturity equal weight:

$$\rho^{\text{yr}} = \frac{1}{28} \sum_{m \in \{5,10\}} \sum_{y=2002}^{2015} \log\left(1 + r_{m,y}^{\text{BCU}}\right) = 0.02597.$$

The resulting discount rate is 2.60 percent per year and is held fixed during estimation.

The OECD average tax wedge, t^{OECD}, measures the tax burden for a single worker without children earning the average wage as a share of the employer's labor cost (OECD, 2026). I use Chile's `AV_TW` series from the `DSD_TAX_WAGES_COU@DF_TW_COU` country dataflow, version 2.1. I recover the model wedge, held fixed during estimation, using

$$\tau = \frac{t^{\text{OECD}}}{1 - t^{\text{OECD}}} = \frac{0.07}{1 - 0.07} = 0.07527.$$

Labor-force exit calibration. I construct the EPS age CDF used to calibrate δ from the same person-month stock sample as the aggregate labor-market moments, comprising men ages 15–64 with valid employment histories and positive sampling weights. Age a is measured continuously in years using age at the start of the spell and elapsed months within it. For each calendar month, I calculate a weighted empirical age CDF. At each observed age, I take the unweighted mean of these CDF values across person-month observations to obtain $\widehat{F}_{\text{age}}(a)$. Each month therefore contributes in proportion to its number of observations at that age.

Let $a_{\min}$ denote the minimum observed age. For ages at which the empirical CDF is below one, the fitted specification is

$$\log\left[1 - \widehat{F}_{\text{age}}(a)\right] = -\delta^{\text{yr}}(a - a_{\min}) + \nu_a,$$

where ν_a is the regression residual. I estimate the slope by weighted least squares without an intercept, weighting each age by its total person-month stock weight, and take its negative as the annual calibration $\widehat{\delta}$. Interpreting the age profile as stationary, I assign this rate to labor-force exit and replacement. The input also absorbs permanent panel attrition, which the data do not separately identify. The regression uses the linear log-survival specification on the restricted sample without an upper-truncation correction.

Time units. One unit of numerical time is one calendar year. I divide empirical durations recorded in months by 12 before evaluating the model profiles. Both the discount-rate calibration and the age-profile slope are already continuous-time annual rates, so $\rho = \rho^{\text{yr}}$ and $\delta = \widehat{\delta}$. Search intensities and separation rates are measured per year. The units of μ_ℓ and σ_ℓ are human capital per year and per square-root year, respectively. Wages, benefits, the recurring formal-state cost, and search costs are annual flows, whereas Ψ_f is a one-time cost. Appendix A.6 gives the transformations under alternative units of time.

Formal-stock wage scale and recovery of A. The empirical formal-stock scale s_f^D is the geometric mean monthly wage over the observed formal months in the wage sample of Appendix B.3. I average component log wages with weights equal to job duration times the survey weight and exponentiate the mean. The sample requires full wage coverage but includes left-censored and incomplete formal spells, subject to the wage sample's age and occupation restrictions. I compute its model counterpart from the equilibrium solved with $A = 1$:

$$s_f^M = a_f \exp\left\{\frac{\int_0^\infty \log x \, m_f(x)\,\mathrm{d}x}{\mathbf{M}_f}\right\}.$$

At positive grid points I evaluate the integral by rectangular quadrature. If the zero node carries formal mass, I retain that mass and approximate the density as constant over its cell $[0, \Delta x]$. The cell's average log human capital is

$$\frac{1}{\Delta x}\int_0^{\Delta x} \log x \,\mathrm{d}x = \log \Delta x - 1.$$

The calculation does not condition on spell completion.

The empirical scale s_f^D is monthly. Matching its annual value to the model scale gives

$$\widehat{A} = \frac{12 s_f^D}{s_f^M}.$$

This implements Equation (17) and expresses A in real 2010 U.S. dollars per year per unit of human capital, using the same solved equilibrium.

I express completed-spell mean wages relative to the formal-stock wage scale. Let $\widehat{L}_{1,\ell}$ denote the stock-weighted empirical mean of the arithmetic spell-average wage in sector ℓ, using fully wage-observed, completed spells that do not begin at the January 1980 left boundary. The corresponding normalized data and model means are

$$\frac{\widehat{L}_{1,\ell}}{s_f^D}, \qquad \frac{L_{1,\ell}^M}{s_f^M}, \qquad \ell \in \{f, i\}.$$

The numerator and scale are monthly in the data and annual in the model, so both ratios are dimensionless.

D Additional Results

D.1 Additional Descriptive Evidence

This subsection collects supplementary wage and mobility evidence. Labor histories and analysis weights are constructed as described in Appendix B.

Wages by Completed Duration. Table D.1 reports the regressions behind Figure 3a. I regress the arithmetic spell-average wage constructed in Appendix B.3 on an intercept and completed duration, separately by sector, using the sector-spell flow weights in Appendix B.2. The figure's markers show weighted mean wages within pooled weighted deciles of duration, while its lines fit the individual spells. Shaded bands are 95 percent confidence intervals for the fitted means, clustered by worker.

For a completed spell of duration T in sector ℓ, integrating the wage dynamics in Section 2.5 gives, for employment paths without reflection at zero,

$$\overline{w}_\ell = w_0 + a_\ell \left[\mu_\ell T/2 + \sigma_\ell \overline{W}_T\right], \quad \overline{W}_T = \frac{1}{T}\int_0^T W_t \, \mathrm{d}t. \tag{D.1}$$

Here, w_0 is the wage at entry.

Taking conditional expectations in Equation (D.1) relates these wage profiles to the model for employment paths without reflection. Brownian increments are measured from spell entry. All expectations below use flow weights and condition on the regression sample, with time and wage units matched to the regression:

$$\mathbb{E}[\overline{w}_\ell \mid T, \ell] = \mathbb{E}[w_0 \mid T, \ell] + a_\ell \left[\frac{\mu_\ell T}{2} + \sigma_\ell \mathbb{E}[\overline{W}_T \mid T, \ell]\right].$$

Completed duration can select both initial wages and shock histories, since search and quitting depend on the evolving human-capital path. In particular, the selected mean shock need not be zero. The fitted slope therefore combines deterministic wage growth with duration-related differences in entry wages and realized shocks.[35]

[35] If a worker remains employed after reaching zero human capital, reflection adds $a_\ell T^{-1}\int_0^T (L_t - L_0)\,\mathrm{d}t$ to Equation (D.1), where $L_t - L_0$ is the regulator accumulated since spell entry. This term vanishes when the worker quits at or before zero.

Table D.1: Spell-average wages and completed duration

	Formal	Informal
Duration (months)	2.92	1.38
	(1.22)	(1.88)
Constant	440.57	393.17
	(26.73)	(26.81)
Spells	3,705	2,372
Workers	2,197	1,639
R^2	0.004	0.001

Notes: Sector-specific flow-weighted regressions of the standardized monthly wage equivalent, in real 2010 USD, on completed duration in months. The sample comprises completed formal and informal sector-spells with valid positive wages throughout, for men ages 15–64 at spell start, beginning directly out of unemployment and lasting at most 36 months. January 1980 starts and spells containing armed-forces or unclassifiable occupations are excluded. SEs clustered by worker are in parentheses. Source: EPS 2002–2015, author's calculations.

Transition Matrices at Alternative Horizons. Table D.2 follows the person-month stock in each origin state and records its state one, six, or twelve months later, conditional on continuous observation. Workers may change states several times, including returning to the origin, before the destination is recorded. A spell's first-exit destination alone does not determine the state observed at the horizon.

Table D.2: Employment-state transition matrices at alternative horizons

Origin state	Formal	Informal	Unemployed
Panel A. One-month horizon			
Formal	99.31	0.20	0.49
Informal	0.62	98.79	0.59
Unemployed	1.71	0.77	97.52
Panel B. Six-month horizon			
Formal	96.76	1.17	2.07
Informal	3.49	94.01	2.50
Unemployed	7.89	3.35	88.76
Panel C. Twelve-month horizon			
Formal	95.06	2.13	2.81
Informal	6.33	90.05	3.62
Unemployed	11.77	4.98	83.25

Notes: Rows report employment state at month t, and columns report employment state at $t + h$. Panels A–C set $h = 1$, 6, and 12 and contain 12,539, 12,538, and 12,536 workers, respectively. Each panel requires continuous observation through its horizon, so intervening gaps are treated as censoring. Entries are row percentages and use person-month stock weights. The sample consists of men ages 15–64 at origin month t; armed-forces occupations are excluded.

Cross-Country Labor-Market Transitions. Table D.3 compares Chile with annual transitions obtained from the continuous-time generators in Bosch & Maloney (2010) for Argentina, Brazil, and Mexico. Under their broad Definition 1 classification, unemployment and time out of the labor force form a common non-employment state, while informal self-employment and informal salaried employment form a common informal state. The foreign state space is therefore (F, INF, N). Chile retains its baseline (F, I, U) classification, with its histories sampled on each country's interview schedule.

The synthetic panels use four interviews six months apart for Argentina, four consecutive monthly interviews repeated one year later for Brazil, and five quarterly interviews for Mexico. Interview timing is anchored at the start of the first non-left-censored spell, and a candidate window is retained only when the worker's state is observed at every required interview. In each of 1,000 Monte Carlo draws, one admissible window is selected uniformly per worker. Annual transitions are pooled using the person-month weight at the origin date and averaged across draws.

Table D.3: Cross-country labor-market states and annual transitions

	Argentina	Brazil	Mexico
Panel A. Aggregate state shares			
Formal	36.8 (54.2)	51.7 (54.1)	55.0 (54.1)
Informal	39.8 (19.0)	28.2 (19.2)	31.5 (19.0)
Non-employment	23.4 (26.8)	20.1 (26.7)	13.5 (26.9)
Panel B. Annual transition probabilities			
$F \to F$	82.1 (94.3)	82.2 (94.1)	82.8 (94.1)
$F \to I$	10.4 (2.4)	9.1 (2.4)	11.8 (2.4)
$F \to U$	7.5 (3.3)	8.7 (3.5)	5.3 (3.5)
$I \to F$	10.9 (7.2)	18.8 (7.1)	22.5 (7.1)
$I \to I$	71.7 (87.8)	67.6 (87.9)	68.9 (87.8)
$I \to U$	17.4 (5.0)	13.6 (5.0)	8.6 (5.1)
$U \to F$	9.6 (10.6)	19.4 (10.5)	17.5 (10.8)
$U \to I$	31.8 (4.7)	21.9 (4.9)	24.3 (4.7)
$U \to U$	58.6 (84.7)	58.6 (84.7)	58.2 (84.5)

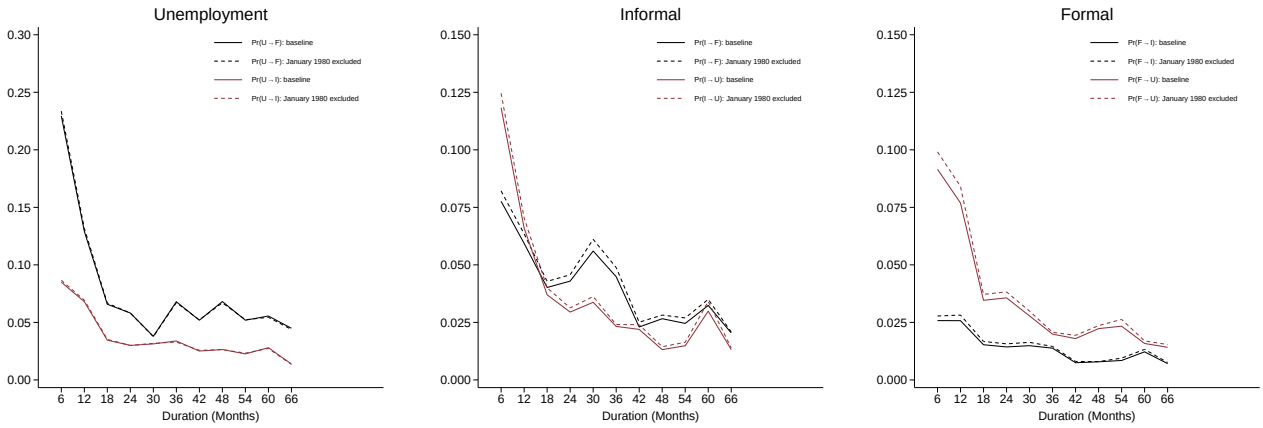
Notes: Entries are percentages. In each cell, the first line is the Definition 1 foreign benchmark and the smaller parenthetical second line is the corresponding country-specific synthetic Chilean weighted Monte Carlo mean. Parentheses do not contain standard errors. The foreign state space is (F, INF, N), while the Chilean state space remains (F, I, U).

D.2 Robustness of the Empirical Evidence

The comparison below examines left-censoring in the empirical evidence constructed from the histories in Appendix B.

Left-Censoring of Sector Spells. Figure D.1 recomputes the estimates after excluding spells beginning in January 1980, which may be left-censored at the beginning of the retrospective history window. The bins, weights, and inference conventions otherwise remain the same.

Figure D.1: Destination-specific hazards excluding January 1980 spells

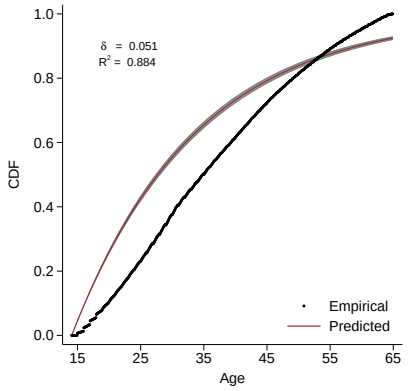


Notes: Solid lines report the baseline destination-specific hazards. Dashed lines exclude sector-spells first observed in January 1980, which are potentially left-censored because the retrospective labor-history window begins in that month. Excluding these spells removes observations whose elapsed duration at the beginning of the history window is unknown. Both specifications use six-month duration intervals and sector-spell flow weights. Panels are organized by origin state.

D.3 Additional Model Results

Age Profile and Constant-Exit Fit. Figure D.2 reports the age profile and descriptive constant-exit fit. Appendix C.3 documents the calibration.

Figure D.2: Age Profile and Fitted Constant-Exit Model



Notes: Black markers report the empirical age CDF for all workers ages 15–64, constructed from person-month stock weights. The solid line is the corresponding descriptive constant-hazard fit, and the shaded region is its 95 percent confidence interval. The model's value of δ is estimated separately on men ages 15–64.