Financial Development, Informality, and Misallocation

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Motivation

- **Labor informality** is a salient feature of many developing economies.
  - **30% - 80%** in Latin America, and similar numbers for Asia, Eastern Europe.
- Middle- and low-income countries have underdeveloped **financial sectors**.
  1. Intermediate private credit to GDP was **31%** for low and middle income countries.
  2. High income countries average **86%** ratio.
- The interaction of **financial frictions** and **labor informality** is unclear.
  1. **Labor informality** creates a buffer for firms to operate in over-regulated economies
  2. **Informal firms** have less access to the **financial sector**.

Research Question

- How do **financial frictions** determine the size of the **informal** sector?
- How do **financial frictions** shape firm dynamics in an **informal** economy?
- Do **financial frictions** lead to misallocation of factors in an **informal** economy?

This Paper

- Document facts about **informality** and **financial frictions** at the firm level.
  1. Static: Relate firm size to **informality** & **financial frictions**.
  2. Dynamic: Relate firm age to **informality** & **financial frictions**.

- Build a general equilibrium model of firm dynamics with:
  1. Firm heterogeneity,
  2. **Informal labor**,
  3. **Financial frictions**.

Road Map

Data and Measurement

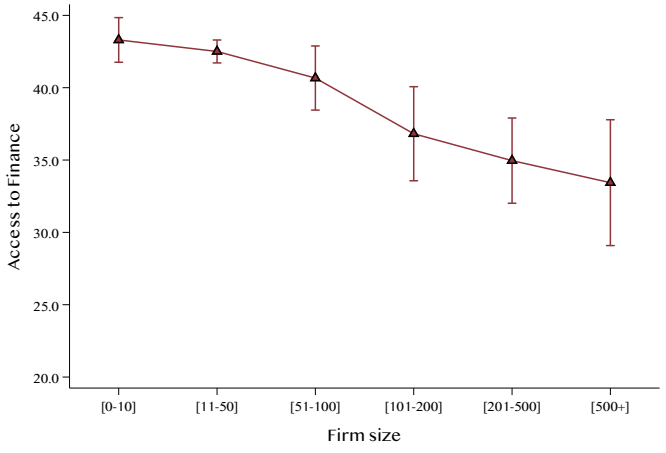
- World Bank Enterprise Surveys (WBES)
  - Repeated cross-section
  - Firm-level survey across 158 countries - 207,000 firms surveyed.
  - Firm characteristics, performance measures
- Relevant variables:
  - **Informality**: Share of workers not reported to tax authorities.
  - **Financial frictions**: Access to banking system (qualitative answer).
  - Firm size: Number of full-time employees.
- IMF **Financial Development Index** (F.D.I)
  - Country-level panel starting on 1980.
  - Accounts for: Financial institutions, markets (size + liquidity), access & efficiency.

Summary Statistics

	Mean	SD	p25	p75	Obs
Access to finance	41.62	34.91	0	75	12,716
Firm size	77.52	376.65	8	44	12,695
Informality	27.88	36.87	0	50	12,716
Firm age	18.67	17.17	7	25	12,716

Notes: Sample contains information for 27 different countries for the year 2006.

Result 1: Large Firms face less Financial Frictions



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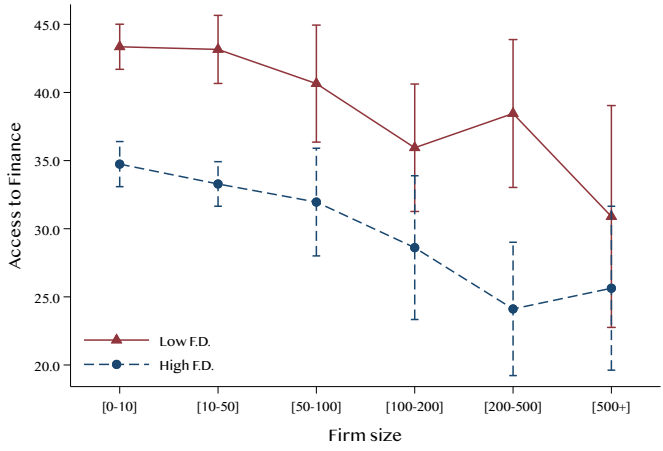
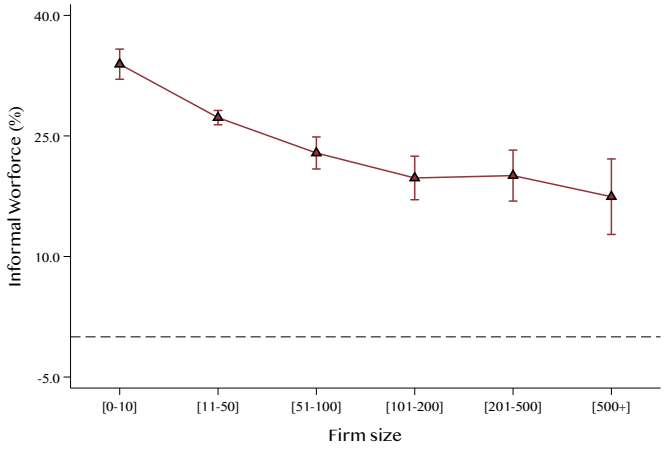


Result 2: Large Firms are less informal intensive



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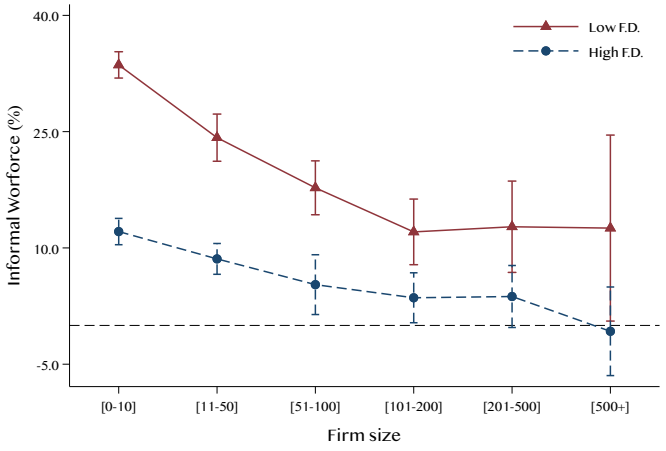

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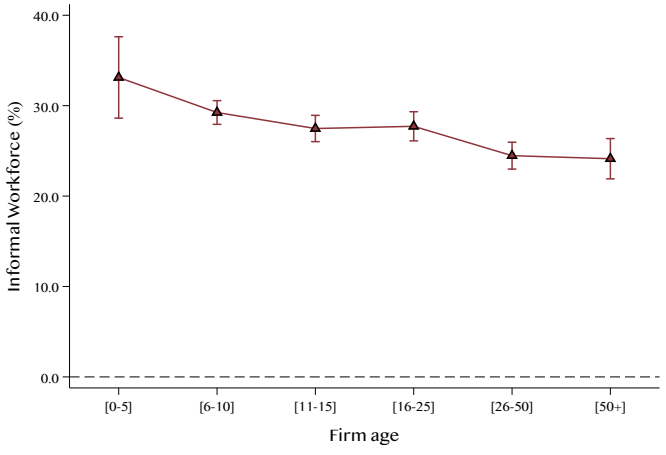

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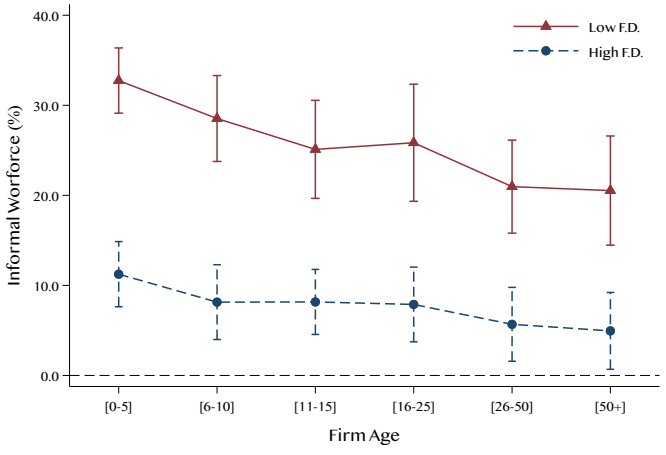

Road Map

Overview

- Firm dynamics model
  - Firms are heterogeneous in productivity
  - Labor is the only factor of production.
  - Firms choose quantity **formal** and **informal** labor.
- Firms face two dynamic problems
  - **Investment** to increase productivity
  - **Accumulation** of research capital.
- Sources of misallocation
  - **Financial frictions** $\longrightarrow$ Borrowing constraint.
  - Tax on **formal** workers
  - Cost to hide **informal** workers.

Households and Preferences

- Time is continuous.
- There is a unit mass of forever-lived households with discount rate ρ.
- Agents have logarithmic preferences over consumption

$$u(c) = \ln(c)$$

- Households choose consumption $c(t)$ and savings $s(t)$.
- The implied Euler equation is

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho$$

Firms: Production

- Firms are heterogeneous in their productivity z
- Choose a bundle of labor $\{\ell_f, \ell_i\}$ to maximize:

$$\pi(z) = \max_{\ell_f, \ell_i \geq 0} \; z^{1-\alpha}(\ell_i + \ell_f)^{\alpha} - w(1+\tau)\ell_f - w\left(1 + \frac{b}{2}\ell_i\right)\ell_i$$

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- Implies a cut-off $\bar{z}$ such that. ▸ Details
  1. Informal Firm: If $z \leq \bar{z}$ then $\ell_f = 0$.
  2. Formal Firm: If $z > \bar{z}$, both ℓ_f and ℓ_i.

Firms: Dynamics

- Firms have a stock of research capital a.
- Productivity follows a Brownian motion

$$\mathrm{d}z(t) = \mu(z(t), a(t)) + \sigma \mathrm{d}W(t)$$

- Investing $\boldsymbol{R}$ units of research capital yields a drift

$$\boldsymbol{\mu} = \left(\frac{2\boldsymbol{R}}{\theta}\right)^{1/2}$$

- Borrowing constraint $\boldsymbol{R} \leq \boldsymbol{\zeta} a$ with $\zeta > 1$.
- Shoe-leather cost to deposit profits d into the research capital account.

Value Function

- The value function of a firm is

$$(r+\delta)V(z,a) = \max_{\substack{0\leq R\leq \zeta a \\ 0\leq d}} \Bigg\{ \pi(z) - d - \frac{\psi}{2}d^2 + V_a(z,a)(r_a a + d - R) + \mu(z,a)V_z(z,a) + \frac{\sigma^2}{2}V_{zz}(z,a) \Bigg\}$$

  with complementary-slackness condition $V(z,a) \geq 0$.

- Death shock δ

Entrants

- There is a mass of entrants m.
- Upon entry, firms pay a fixed cost c_e and
  1. Get a productivity draw $z_0 \sim F_0(z)$.
  2. Have no initial stock of research capital $a_0 = 0$.
- Free entry condition implies

$$m = \bar{m} \exp\left(\varepsilon \int_0^\infty V(z_0, 0) \mathrm{d}F_0(z_0) - c_e\right)$$

where ε is the elasticity of new entrants.

Equilibrium

Given government policies τ, an Equilibrium is a tuple of policy functions (p.f)

$$\left\{\ell_f(z,a), \ell_i(z,a), d(z,a), V(z,a), \mu(z,a), R(z,a)\right\}$$

and prices $\{r, r_a, w\}$ plus a density function $g(z,a)$ such that

1. Taking prices (r, r_a, w) as given, the p.f solve the value function $V(z,a)$.
2. Taking the p.f $\mu(z,a)$ and $d(z,a)$ as given, $g(z,a)$ solves the KFE. KFE
3. Taking the p.f and the density function $g(z,a)$ as given, prices adjust so that
   - Labor market clears,
   - The market for research capital clears
4. The government has a balanced budget.

Thank You

Appendix

Static Solution

- The cut-off is described by the equation

$$\bar{z} = \left(\frac{w(1+\tau)}{\alpha} \right)^{\frac{1}{1-\alpha}} \frac{\tau}{b}$$

- Optimal labor choices are

$$\ell_i = \begin{cases} \ell_i^*(z) & \text{if } z < \bar{z} \\ \frac{\tau}{b} & \text{if } z \geq \bar{z} \end{cases} \qquad \ell_f = \begin{cases} 0 & \text{if } z < \bar{z} \\ z \left(\frac{\alpha}{w(1+\tau)} \right)^{\frac{1}{1-\alpha}} - \frac{\tau}{b} & \text{if } z \geq \bar{z} \end{cases}$$

where $\ell_i^*(z)$ is implicitly defined by the following equation

$$\frac{\alpha z^{1-\alpha}}{w} = (1 + b\ell_i^*(z))\, \ell_i^*(z)^{1-\alpha}.$$

Back

Kolmogorov Forward Equation

- The Kolmogorov Forward Equation (KFE) for $z \geq z_0$

$$\begin{aligned}\frac{\partial g(z,a,t)}{\partial t} &= -\partial_a[d(z,a)g(z,a)] \\ &\quad - \partial_z[\mu(z,a)g(a,z)] \\ &\quad + \frac{1}{2}\partial_{zz}\left[\sigma^2(z)g(z,a)\right] \\ &\quad - \delta g(z,a) = 0\end{aligned}$$

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